The Impact of Integrated Analysis on Supply Chain Management: A Coordinated Approach for Inventory Control Policy

G. Reza Nasiri, Hamid Davoudpour, Behrooz Karimi

Department of Industrial Engineering, Amirkabir University of Technology, Tehran, Iran

reza_nasiri@aut.ac.ir, hamidp@aut.ac.ir, b.karimi@aut.ac.ir

Abstract

Purpose – The purpose of the paper is to integrated model for the location of warehouse, the allocation of retailers to the opened warehouses, and finding perfect policy for inventory control to managing order quantity and safety stock level. The goal is to select the optimum numbers, locations, capacities of the opening warehouses and inventory policy so that all stochastic customer demands to be satisfied.

Design/methodology/approach – The model assumes that the location of plant has already been determined and answers the following questions: what is the number of warehouses to open? How retailers are allocated to the plant? What are the optimum capacities for the opening warehouses? What is the best inventory policy for this supply chain? What are the total minimum costs?

Findings- The model was formulated as a non-linear mixed integer programming and solved using Lagrange relaxation and sub-gradient search for the location/allocation module and a procedure for the capacity planning module. The results for the randomly selected problems show that the average gap duality ranges are between 0.51 and 1.02 percent. Also, from the CPU time point of view, the performance was very good.

Originality/value- An attempt is made to integrate location, allocation, and inventory decisions in the design of a supply chain distribution network.

Keyword Supply chain management, Distribution planning decisions, Inventory control policy

Paper type Research paper

1. Introduction

The distribution planning decision (DPD) is one of the most comprehensive strategic decision issues that needs to be optimized for the long-term efficient operation of the whole Supply Chain (SC). The DPD involves optimizing the transportation plan for allocating goods and/or services from a set of sources to various destinations in a supply chain (Liang, 2006). An important strategic issue related to the design and operation of a physical distribution network in a supply chain system is the determination of the best sites for intermediate stocking points, or warehouses. The use of warehouses provides a company with flexibility to respond to changes in the marketplace and can result in significant cost savings due to the economies of scale in transportation or shipping costs.

In a supply-chain, a number of organizations cooperate with each other in order to improve the competitive capabilities of the whole chain. Among the typical processes of a supply chain, distribution corresponds to the flow of materials and goods between an organization and its suppliers or customers. Designing distribution networks has attracted the attention of many researchers during recent years. Satisfying the customers' demands on time is very important, for being cost-effective, and for its role in increasing the service level of customers.

The purpose of this paper is to design an integrated distribution center location, allocation and inventory decisions for multi-commodity supply chains in a stochastic environment. The paper has two important applied and theoretical contributions. First, it presents a new comprehensive and practical, but tractable, optimization model for distribution network designing. Moreover, the model incorporates optimizing the inventory policy in to the facility location decisions. And second, it introduces a novel solution approach based on the

Lagrangian Relaxation (LR) heuristic, improved with an efficient heuristic to solve the complex problems.

The rest of the paper is organized as follows. The relevant literature is reported in Section 2. In Section 3, the problem is defined more precisely and a mathematical formulation of the distribution design problem is presented. The proposed LR and its mechanisms' are explained in Section 4. The structure of test problems and corresponding computational results are discussed in Section 5. Finally, in Section 6, concluding remarks are provided and some directions for future research are proposed.

2. Literature Review

In the previous studies, deterministic, stochastic and fuzzy customers' demands have been considered, but more attention has been paid to the deterministic cases (Farahani and Elahipanah, 2008). Generally, one of the main constraints in modeling such distribution networks are the capacity constraints (Miranda and Garrido, 2008). In some cases, in addition to the capacity constraints, some other restrictions, e.g. the amount of covered demands and the service levels of the warehouses are also considered. Service level can be also stated based on the capacities of the warehouses. The permissible number of facilities to be opened is another constraint used in some Facility Location Problems (FLP). The distribution network models consist of one or more objective functions. Chan et al. (2005), Liang (2006), Melachrinoudis et al. (2005) and Sabri and Beamon (2000) applied cost minimization and service level maximization, simultaneously. In some cases, setting up a balance between the facilities of different levels of the supply chain has been taken into account. Maximizing the robustness of the decisions is another objective recently considered and developed by researchers (Chen and Lee, 2004). An example would be an objective function maximizing the balance between the distribution centers, regarding the total distance of transportation to the retailers.

Many researchers have extensively studied facility and demand allocation problems. Previous research studies have been well surveyed by Brown et al. (1987), Hurter and Martinich (1989), Cohen and Moon (1990), Pirkul and Jayaraman (1998), Melachrinoudis and Min (2000), Nozick (2001), Eksioglu et al. (2006). Recently some authors have incorporated inventory control decisions into FLP. For example, Miranda and Garrido (2004, 2008), Daskin et al. (2002) and Shen et al. (2003), present similar versions of a FLP model incorporating the inventory control decisions. In these works, the ordering decisions are based on the Economic Order Quantity (EOQ) model. Those model structures assume normality and independency for the demand pattern. They greatly differ though, in the solution methods. Indeed, while Daskin et al. (2002) and Miranda and Garrido (2004) apply different versions of the LR method, Shen et al. (2003) reformulate the problem as a Set Covering Problem, which is then solved through a hybrid heuristic mixing columns generation and branch and bound methods. In Shen et al. (2003) and Daskin et al. (2002) studies, the clients represent retailers, each of which is a potential candidate for a distribution center. In Miranda and Garrido (2004), each client represents a cluster of final demand entities. Furthermore, this study presents a numerical evaluation of the benefits of this simultaneous approach (inventory location decisions), instead of the traditional sequential approach, in which and inventory control decisions are make independently.

The literature presents different approaches to solve the FLP models. For example, Pirkul and Jayaraman (1996, 1998), Nozick (2001), Syam (2002), Miranda and Garrido (2004, 2008), Eskigun et al. (2005) and Amiri (2006) apply Lagrangian Relaxation in addition to the basic imbedded heuristics to obtain feasible solutions at each iteration. Chung et al. (1992) solve a FI[illegible]l considering the dual problem of a set of linear relaxations.

Hwang (2002), Syarif et al. (2002), Zhou et al. (2002), Chan et al. (2005) and Altiparmak et al. (2006) applies genetic algorithms to solve the FLP.

In this paper, we classify distribution problem based on three following points of view (Fig. 1):

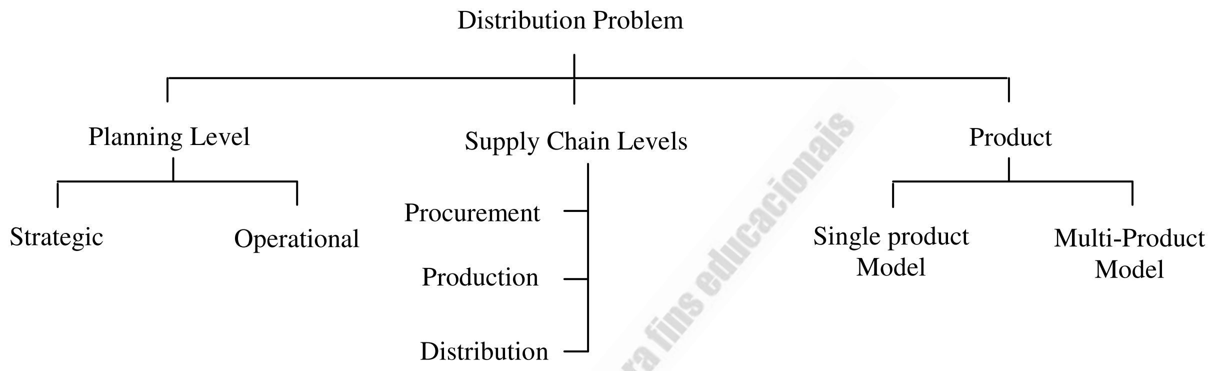


Fig. 1. Classification of Distribution Problems in Supply Chain Management

- Planning Level: Strategic or Operational.
- Supply Chain Levels: Procurement, Production or Distribution.
- Product condition: Single Product model or Multiple Product model.

Thereby, summary of previous models developed to the distribution problems is shown in Table 1.

Table 1. Classification of Distribution Problems

Planning Level	Strategic	Brown et al. (1987), Cohen and Moon (1990), Pirkul and Jayaraman (1996, 1998), Melachrinoudis and Min (2000), Zhou et al. (2002), Eskigun et al. (2005), Altiparmak et al. (2006), Amiri (2006)
	Operational	Que et al. (1999), Wang et al. (2004), Chan et al. (2005)
	Strategic/ Operational	Sabri and Beamon (2000), Jayaraman and Pirkul (2001), Nozick (2001), Hwang (2002), Syarif et al. (2002), Jayaraman and Ross (2003), Chen and Lee (2004), Miranda and Garrido (2004), Melachrinoudis et al. (2005), Miranda and Garrido (2008)
Supply Chain Levels	Procurement	
	Production	Wang et al. (2004), Chan et al. (2005)
	Distribution	Brown et al. (1987), Pirkul and Jayaraman (1998), Nozick (2001), Zhou et al. (2002), Eskigun et al. (2005), Amiri (2006)
	Production/ Distribution	Cohen and Moon (1990), Pirkul and Jayaraman (1996), Melachrinoudis and Min (2000), Hwang (2002), Jayaraman and Ross (2003), Chen and Lee (2004), Miranda and Garrido (2004), Melachrinoudis et al. (2005), Nonino and Panizzolo (2007), Miranda and Garrido (2008)
	Procurement/ Distribution	Que et al. (1999)
	Procurement/ Production/ Distribution	Sabri and Beamon (2000), Jayaraman and Pirkul (2001), Syarif et al. (2002), Altiparmak et al. (2006), , Nagar and Jain (2008)
Product	Single product model	Nozick (2001), Hwang (2002), Syarif et al. (2002), Miranda and Garrido (2004), Chan et al. (2005), Melachrinoudis et al. (2005), Altiparmak et al. (2006), Amiri (2006), Miranda and Garrido (2008)
	Multi-product model	Brown et al. (1987), Cohen and Moon (1990), Pirkul and Jayaraman (1996, 1998), Que et al. (1999), Melachrinoudis and Min (2000), Sabri and Beamon (2000), Jayaraman and Pirkul (2001), Jayaraman and Ross (2003), Chen and Lee (2004), Wang et al. (2004), Eskigun et al. (2005)

One major drawback in most of past research studies like Gourdin et al. (2000), Jayaraman (1998), Pirkul and Jayaraman (1998), Tragantalerngsak et al. (2000) is that they limit the number of capacity levels available to each facility to just one (Amiri, 2006). However, as it is the case in practice, there usually exist several capacity levels to choose from for each facility. The use of different capacity levels makes the problem more realistic and, at the same time, more complex to be solved. Another major drawback in some previous studies is that the nu[illegible]d to a pre-specified value is limited. Moreover, these studies fail to describe how this value can be determined in advance. Amiri (2006) represents a significant improvement over past research by presenting a unified model of the problem which includes the numbers, locations, and capacities of both warehouses and plants as variables to be determined in the model; and, at the same time develops the best strategy for distributing the product from the plants to the warehouses and from the warehouses to the customers. He developed an efficient heuristic solution procedure based on Lagrangian Relaxation of the problem, and reported the extensive computational tests with up to 500 customers, 30 potential warehouses, and 20 potential plants. The distinctions between our research and the work of Amiri (2006) are 1) in our model in which the distribution network has a single plant, and 2) we also incorporate the tactical/operational decisions into the facility location problem solution scheme. Specifically, inventory decisions will be simultaneously modeled with the distribution network design. This inclusion acquires an especial relevance in the presence of high holding costs (e.g. frozen food industry) and high-variability demands.

In this paper, a multi-product, multi-echelon location-allocation model for the optimization of a supply chain design is proposed. This model integrated the inventory decisions into distribution network design with stochastic market demands.

However, large-size problems of the resulting non-linear mixed-integer programming model cannot be solved using the exact methods in a reasonable time. Therefore, a Lagrangian Relaxation algorithm is designed to solve the large-size problems. In order to verify the performance of the proposed LR, the results obtained from solving the small-size problems are compared with the results obtained from LINGO optimization software.

3. Problem Description and Formulation

In this paper, we consider the problem of designing a supply chain distribution network which in es of warehouses (the plant location is known and fixed) and the best strategy for distributing each product from the plant to the warehouses and from the warehouses to the customers simultaneously; Each of these demands has a stochastic demand, in which d_{il} and v_{il} denote the mean and variance retailer demand i for product l, respectively (Figure 2). Besides the common total cost objective function elements, e.g. warehouse establishment cost, ordering and transportation costs, another component of objective function which should be considered is the safety stock holding cost.

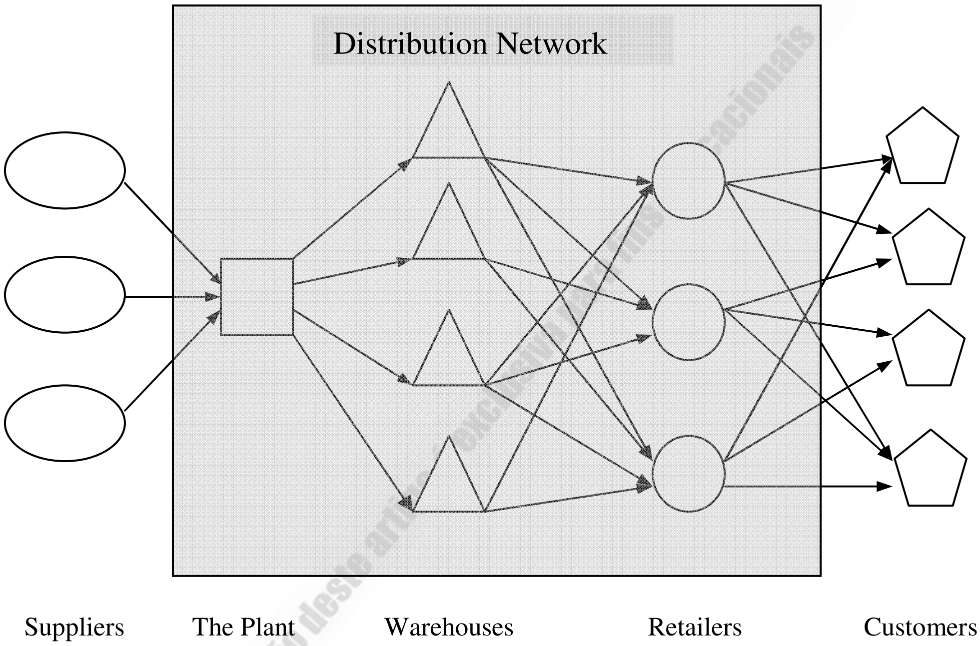


Fig. 2. Stricture of the supply chain distribution network

The following notations are used for the mathematical formulation of our model:

Indices:

I index set of customers/customer zones
J index set of potential warehouse sites
L index set of products
H index set of capacity levels available to the potential warehouses

Parameters:

TC_{ijl} unit cost of supplying product l to customer zone i from warehouse on site j
$\overline{TC}_{jl}$ unit cost of supplying product l to warehouse on site j from the plant
T_{jl} the elapsed time between two consecutive orders of product l for site j
F_{jh} fixed cost per unit of time for opening and operating warehouse with capacity level h on site j
d_{il} mean demand per time unit of product l from customer zone i

v_{il}	variance of the demand per unit product l from customer zone i
HC_{jl}	holding cost per time unit of product l in warehouse on site j
OC_{jl}	ordering cost of product l from warehouse on site j to the plant
cap_{jh}	capacity of warehouse on site j with capacity level h
s_l	space requirement of product l at any warehouse
PH	planning horizon

Decisions variables:

X_{jh}	it takes value 1, if a warehouse with capacity level h is installed on potential site j, and 0 otherwise
Y_{ijl}	it takes value 1, if the warehouse on site j serves product l of customer i, and 0 otherwise
D_{jl}	mean demand per time unit of product l to be assigned to warehouse on site j
V_{jl}	variance of the demand per time unit of product l to be assigned to warehouse on site j

3.1. Calculating of the total system cost

In this Section we used the Weber problem for modeling allocation/transportation costs (Drezner et al., 2002). The objective of the allocation function is:

$$F(Y_{ijl}) = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L} w_{il} d(X_j, P_i) Y_{ijl} \tag{1}$$

where w_{il} is the weights of the demand customer zone i for product l and $d(X_j, P_i)$ is the distance between customer zone i, located at $P_i = (a_i, b_i)$ and the warehouse j located at $X = (x_j, y_j)$. We consider the following Euclidean distance measures:

$$d(X_j, P_i) = \sqrt{(x_j - a_i)^2 + (y_j - b_i)^2} \tag{2}$$

Since w_{il} is a stochastic parameter then the allocation cost function which should be minimized is:

$$Min\, E\left[F\left(Y_{ijl}\right)\right] = Min_x\, E_w\left[\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L} w_{il} d(X_j, P_i) Y_{ijl}\right]$$

$$= Min_x \sum_i \sum_j \sum_l E\left(w_{il}\right) d(X_j, P_i) Y_{ij;} \tag{3}$$

For calculating the transportation costs we replaced the distance component in the above objective function with the transportation cost in the whole network. Now we can formulate the transportation costs objective function which is equivalent to:

$$Min\ Z_{TC} = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L} (\overline{TC}_{jl} + TC_{ijl}) \cdot d_{il} \cdot Y_{ijl} \tag{4}$$

where $E(w_{il}) = d_{il}$.

For calculating the inventory holding cost at any located warehouse, we consider the continues inventory revision (Miranda & Garrido, 2004). In this inventory control policy, when the inventory level of product l falls below r_{jl}, an order of Q_{jl} units is triggered which is received after LT_{jl} time units. Figure 3 shows the stochastic demand pattern and warehouse fulfill rate. In this figure continues line is the on hand inventory and the segmented line is the inventory position.

If an order is submitted for any located warehouse, the inventory level must cover the customers' demand during lead time LT_{jl} , with a given probability $1-\alpha$ from the DMs. This probability is known as the service level of the inventory system. The service level constraint can be written as follows:

$$\text{Prob}\,(D(LT_{jl}) \leq D_{\max(jl)}) = 1-\alpha \tag{5}$$

where $D(LT_{jl})$ is the uncertain demand, assigned to the warehouse j during the lead time for product l and $D_{\max(jl)}$ is maximum demand during the lead time and can be expressed as follow:

$$D_{\max(jl)} = \overline{D}_{jl} + ss_{jl} \tag{6}$$

where $\overline{D}_{jl}$ is the mean demand, assigned to warehouse j during the lead for product l and ss_{jl} is the level of safety stock inventory that should be held at warehouse j for the product l. If we assume a Normal distribution demand, and consider $D_{\max(jl)}$ as the reorder point, r_{jl} can be determined as follows:

$$r_{jl} = E(D_{jl})\cdot E(LT_{jl}) + Z_{1-\alpha}\cdot\sqrt{(E[D_{jl}])^2\sigma^2_{LT_{JL}}}\cdot\sqrt{E(LT_{jl})V_{jl}} \tag{7}$$

Since in this paper, we assume that the LT_{jl} is a constant parameter, then Eq. (A7) is:

$$r_{jl} = D_{jl}\cdot LT_{jl} + Z_{1-\alpha}\cdot\sqrt{V_{jl}}\cdot\sqrt{LT_{jl}} \tag{8}$$

where $Z_{1-\alpha}$ is the value of the Standard Normal distribution, which accumulates a probability of $1-\alpha$. This parameter is assumed fixed for the entire network, determining a uniform service level of the system.

The average holding cost rate for each warehouse j and product l (\$/day) based on the Eq. (8), can be written as:

$$HC_{jl}\cdot Q_{jl}/2 + HC_{jl}\cdot Z_{1-\alpha}\cdot\sqrt{LT_{jl}}\cdot\sqrt{V_{jl}} \tag{9}$$

The first term of the Eq. (9) is the average cost incurred due to the holding the order quantity Q_{jl}, which is the inventory of product l used to cover the demand arisen during two successive orders. The second term in (9) is the average cost associated with the safety stock kept at the warehouse j $(Z_{1-\alpha}\cdot\sqrt{LT_{jl}}\cdot\sqrt{V_{jl}})$.

In this case we assume that there is not any capacity constraint for the order quantity. So, differentiating the objective function in terms of Q_{jl} for each warehouse and product, and eq

$$\frac{HC_{jl}}{2} - \frac{OC_{jl}}{Q_{jl}^2} \cdot D_{jl} = 0 \tag{10}$$

From Eq. (10), we could obtain:

$$Q_{jl}^* = \sqrt{\frac{2 \cdot OC_{jl} \cdot D_{jl}}{HC_{jl}}} \qquad \forall j = 1,...,J \quad , \quad \forall l = 1,...,L \tag{11}$$

By replacing Eq. (11) in Eq. (9), the objective function can be expressed as follows:

$$Min\ Z_2 = \sum_{j=1}^{J}\sum_{h=1}^{H} F_{jh} \cdot X_{jh} + PH \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L} (\overline{TC}_{jl} + TC_{ijl}) \cdot d_{il} \cdot Y_{ijl}$$

$$+ PH \sum_{j=1}^{J}\sum_{l=1}^{L} \sqrt{2 \cdot HC_{jl} \cdot OC_{jl}} \cdot \sqrt{D_{jl}} + PH \sum_{j=1}^{J}\sum_{l=1}^{L} HC_{jl} \cdot Z_{1-\alpha} \cdot \sqrt{LT_{jl}} \cdot \sqrt{V_{jl}} \tag{12}$$

3.2. Formulation of MDNDMC

In this study, warehouses can have multi-level capacity. So, for adapting warehouse capacity with the sum of customer demands which are assigned to this warehouse, the capacity planning is considered in this paper. The multi-product distribution network design with the multi-level capacity (MDNDMC) can be summarized as follows:

$$Min \quad \sum_{j=1}^{J}\sum_{h=1}^{H} F_{jh} \cdot X_{jh} + \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L} PH \cdot (\overline{TC}_{jl} + TC_{ijl}) \cdot d_{il} \cdot Y_{ijl}$$

$$+ \sum_{j=1}^{J}\sum_{l=1}^{L} PH \cdot \sqrt{2 \cdot HC_{jl} \cdot OC_{jl}} \cdot \sqrt{D_{jl}} + \sum_{j=1}^{J}\sum_{l=1}^{L} PH \cdot HC_{jl} \cdot K \cdot \sqrt{LT_{jl}} \cdot \sqrt{V_{jl}} \tag{13}$$

Subject to:

$$\sum_{j=1}^{J} Y_{ijl} = 1 \qquad \forall i = 1,...,I \quad , \quad \forall l = 1,...,L \tag{14}$$

$$\sum_{i=1}^{I}\sum_{l=1}^{L} d_{il} \cdot s_l \cdot Y_{ijl} \le \sum_{h=1}^{H} cap_{jh} \cdot X_{jh} \qquad \forall j = 1,...,J \tag{15}$$

$$\sum_{i=1}^{I} d_{il} \cdot Y_{ijl} = D_{jl} \qquad \forall j = 1,...,J \ , \ \forall l = 1,...,L \tag{16}$$

$$\sum_{i=1}^{I} v_{ijl} \cdot Y_{ijl} = V_{jl} \qquad \forall j = 1,...,J \ , \ \forall l = 1,...,L \tag{17}$$

$$\sum_{h=1}^{H} X_{jh} \leq 1 \qquad \forall j = 1,\ldots,J \tag{18}$$

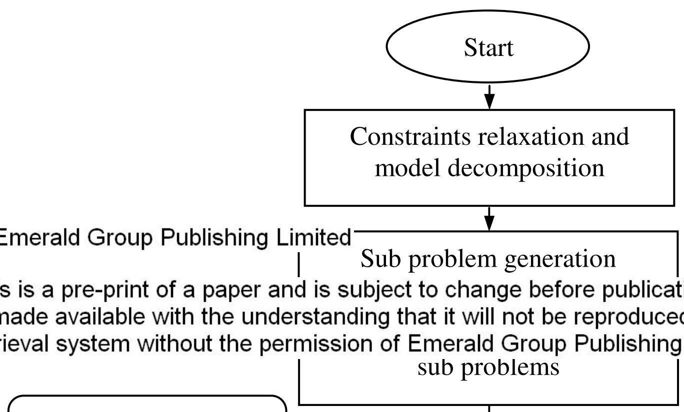
$$X_{jh}, Y_{ijl} \in \{0,1\} \quad \forall i = 1,\ldots,I\ ,\ \forall j = 1,\ldots,J\ ,\ \forall l = 1,\ldots,L\ ,\ \forall h = 1,\ldots,H \tag{19}$$

Eq. (14) assures that each retailer is served exactly for each product by one warehouse (single source). Eq. (15) represents the warehouse capacity level (only if the warehouse is installed). Eq. (16) computes the served average demands by the warehouse *j*. Eq. (17) computes the total variance of the served demand by warehouse *j*. Implicitly, we assume that the demands are independently distributed across the retailers, thus all the covariance terms are zero. Eq. (18) ensures that each warehouse can be opened at only one capacity level. Finally, Eq. (19) states the integrality for the variables X_{jh} and Y_{ijl}.

The Lagrangian relaxation and solution method presented in the next sections can also be easily modified to handle the extended model.

4. Solution Approach

Problem MDNDMC is a mixed zero-one non-linear model. We have used LINGO 8.0 software to solve the small size problems. But, we need an efficient and effective solution method to obtain the optimal or near-optimal solutions, particularly for the medium and large-scaled problems within a reasonable time. Thus, in order to solve the industrial real-size problems, an approach based on the Lagrangian Relaxation and sub-gradient method is proposed which is summarized in Figure 4.



The cost parameters are expressed as follows:

$$CT_{ijl} = PH \cdot (\overline{TC}_{jl} + TC_{ijl}) \cdot d_{il}\ ,\ CH_{jl} = PH \cdot \sqrt{2 \cdot HC_{jl} \cdot OC_{jl}}\ ,$$

$$CS_{jl} = PH \cdot HC_{jl} \cdot K \cdot \sqrt{LT_{jl}}$$

So, the objective function can be written as follows:

$$\sum_{j=1}^{J}\sum_{h=1}^{H} F_{jh} \cdot X_{jh} + \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L} CT_{ijl} \cdot Y_{ijl} + \sum_{j=1}^{J}\sum_{l=}^{L} CH_{jl} \cdot \sqrt{D_{jl}} + \sum_{j=1}^{J}\sum_{l=1}^{L} CS_{jl} \cdot \sqrt{V_{jl}} \qquad (20)$$

4.1. A Lagrangian Relaxation to Problem MDNDMC

We propose a LR formulation for the MDNDMC, incorporating two additional constraints into the model, given by (see Miranda & Garrido, 2004):

$$\sum_{j=1}^{J} D_{jl} \le DT_l = \sum_{i=1}^{I} d_{il} \qquad \forall l = 1,\ldots,L \qquad (21)$$

$$\sum_{j=1}^{J} V_{jl} \le VT_{jl} = \sum_{i=1}^{I} v_{il} \qquad \forall l = 1,\ldots,L \qquad (22)$$

Constraint (21) states that the total average of demand for each product assigned to warehouses does not exceed the total average demand of the retailers (customers) for that product. Constraint (22) shows that the total variance of demand for each product assigned to the warehouses does not exceed the total variance demand of the retailers for that product. These constraints are added to solve the sub-problems. Furthermore, Constraints (16) and (17) are replaced by:

$$\sum_{i=1}^{I} d_{il} \cdot Y_{ijl} \leq D_{jl} \qquad \forall j = 1,...,J \ , \ \ \forall l = 1,...,L \tag{23}$$

$$\sum_{i=1}^{I} v_{il} \cdot Y_{ijl} \leq V_{jl} \qquad \forall j = 1,...,J \ , \ \ \forall l = 1,...,L \tag{24}$$

We obtain the Lagrangian Relaxation of the problem by dual constraints (14), (23) and (24), using multipliers α_{il}, β_{il} and γ_{jl} for all $i \in I$, $j \in J$, $l \in L$, respectively. In this manner, the relaxed model can be written as follows:

Problem LR:

$$Z_{LR} = Min \qquad \sum_{j=1}^{J}\sum_{l=1}^{L}(CH_{jl} \cdot \sqrt{D_{jl}} - \beta_{jl} \cdot D_{jl}) + \sum_{j=1}^{J}\sum_{l=1}^{L}(CS_{jl} \cdot \sqrt{V_{jl}} - \gamma_{jl} \cdot V_{jl})$$

$$+\sum_{j=1}^{J}\sum_{h=1}^{H} F_{jh} \cdot X_{jh} + \sum_{I=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L}(CT_{ijl} + \beta_{jl} \cdot d_{il} + \gamma_{jl} \cdot v_{il} - \alpha_{il}) \cdot Y_{ijl} + \sum_{i=1}^{I}\sum_{l=1}^{L}\alpha_{il}$$

Subject to: (15), (18), (19), (21) and (22)

(25)

The optimization algorithm must solve the relaxed problem for a given set of Lagrangian multipliers in an iterative structure, which is explained in the next Section.

4.2. Solution Procedures for Sub-Problems

The relaxed MDNDMC can be decomposed and written as independent sub-problems giving the values of the multipliers from the Lagrangian dual problem. The Lagrangian Relaxation of MDNDMC involves three types of sub-problems in which the first two sub-problems at the iteration k are as follows:

$$SP1^k \quad Min \quad \sum_{j=1}^{J}\sum_{l=1}^{L}(CH_{jl} \cdot \sqrt{D_{jl}} - \beta_{jl}^{k} \cdot D_{jl})$$

$$\text{Subject to:} \quad \sum_{j=1}^{J} D_{jl} \le DT_l \,, \quad \forall l = 1,\ldots,L \tag{26}$$

$$D_{jl} \ge 0 \quad \forall j = 1,\ldots,J, \quad \forall l = 1,\ldots,L$$

$$SP2^k \quad Min \quad \sum_{j=1}^{J}\sum_{l=1}^{L}(CS_{jl} \cdot \sqrt{V_{jl}} - \gamma_{jl}^{k} \cdot V_{jl})$$

$$\text{Subject to:} \quad \sum_{j=1}^{J} V_{jl} \le VT_l \qquad \forall l = 1,\ldots,L \tag{27}$$

$$V_{jl} \ge 0 \qquad \forall j = 1,\ldots,J \;, \quad \forall l = 1,\ldots,L$$

$SP1^k, SP2^k$ sub-problems can be solved through a method proposed by Miranda and Garrido (2004).

The third sub-problem is a Capacitated Facility Location Problem (CFLP) with multi-level capacity for each warehouse. Each warehouse can be assigned at most one capacity level. This sub-problem is as follows:

$$SP3^k \quad Min \quad \sum_{j=1}^{J}\sum_{h=1}^{H} F_{jh} \cdot X_{jl} + \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L}(CT_{ijl} + \beta_{jl}^{k} \cdot d_{il} + \gamma_{jl}^{k} \cdot v_{il} - \alpha_{il}^{k}) \cdot Y_{ijl} \tag{28}$$

$$\text{Subject to:} \quad \sum_{i=1}^{I}\sum_{l=1}^{L} d_{il} \cdot s_l \cdot Y_{ijl} \le \sum_{h=1}^{H} cap_{jh} \cdot X_{jh} \qquad \forall j = 1,\ldots,J \tag{29}$$

$$\sum_{h=1}^{H} X_{jh} \le 1 \qquad \forall j = 1,\ldots,J \tag{30}$$

$$X_{jh}, Y_{ijl} \in \{0,1\} \quad \forall i = 1,\ldots,I, \; \forall j = 1,\ldots,J, \; \forall l = 1,\ldots,L, \; \forall h = 1,\ldots,H \tag{31}$$

For each warehouse j at each capacity level h, we can decompose $SP3^k$ into $J \times H$ sub-problems,. For these sub-problems, X_{jh} is either equal to 0 or 1. Considering Eq. (29), if X_{jh} is equal to 0, then Y_{ijl} is equal to 0 for all i, j& l indices, otherwise, if X_{jh} is equal to 1, then the sub-problem becomes a 0-1 knapsack problem. The 0-1 knapsack problem is as follows:

$$SP3-1^k\,(j) \quad Min \quad V_{jh} = \sum_{i=1}^{I}\sum_{l=1}^{L}(CT_{ijl} + \beta_{jl}^k \cdot d_{il} + \gamma_{jl}^k \cdot v_{il} - \alpha_{il}^k) \cdot Y_{ijl} \tag{32}$$

$$\text{Subject to:} \qquad \sum_{i=1}^{I}\sum_{l=1}^{L} d_{il} \cdot s_l \cdot Y_{ijl} \le cap_{jh} \tag{33}$$

$$Y_{ijl} \in \{0,1\} \quad \forall i = 1,\ldots,I, \quad \forall l = 1,\ldots,L \tag{34}$$

Note that for each candidate location j and each capacity level h, the above model must be solved, so summation on j and h is not necessary in the objective function and constraints.

Using an exact solution algorithm, the *SP3-1(j)* sub-problem yields a tighter lower bound on the MDNDMC optimal value but needs a significant computational time when dealing with a real problem instance. Hence, we use Dantzig's (1957) upper bound on the objective function value of a *SP3-1(j)* sub-problem. The resulting lower bound on the MDNDMC optimal objective function value is less tight, but saves significant computing time. The Dantzig bound is considered to be very tight for the knapsack problems and its worst-case performance ratio is computed as 0.5 (Dantzig, 1957).

This algorithm executes for any potential warehouses and the demands are assigned to these warehouses on the basis of the mentioned algorithm. When the whole V_{jh} values are computed, the X_{jh} value is then calculated by solving the following optimization problem:

$$SP3-2^k\,(j) \quad Min \quad \sum_{j=1}^{J}\sum_{h=1}^{H}(F_{jh} + V_{jh}) \tag{35}$$

Subject to: $$\sum_{i=1}^{I}\sum_{l=1}^{L} d_{il} \cdot s_l \le \sum_{j=1}^{J}\sum_{h=1}^{H} cap_{jh} \cdot X_{jh} \tag{36}$$

$$\sum_{h=1}^{H} X_{jh} \le 1 \qquad \forall j = 1,\dots,J \tag{37}$$

$$X_{jh} \in \{0,1\} \qquad \forall j = 1,\dots,J \;,\; \forall h = 1,\dots,H \tag{38}$$

The objective function is the sum of location and allocation costs. Constraint (36), assures that the capacity of the opened warehouses is enough to satisfy the total demand of customers. Constraints (37) and (38) are the same as constraints (18) and (19), respectively. Note that the location and allocation decisions are made by solving the *SP3-1* & *SP3-2* sub-problems. For the location decision, the values of $(F_{jh} + V_{jh})$ for each candidate locations in all capacity levels are calculated and sorted in ascending order. The X_{jh} values are equal to one unless the capacity of the selected warehouse becomes less than the total space requirement for the total demand of customers. The solution of the third sub-problem will be finished by the replacement of $Y_{ijl} = 0$ for all warehouses that are not selected.

At first, the procedure determines the total capacity of open warehouses to check if there is enough capacity to satisfy total demand. If not, the total available capacity can be increased by either opening a warehouse that was not selected in the solution of problem SP3-2, or increasing the capacity of an open warehouse to the next higher level.

It is noted that Y_{ijlh} assignment matrix is now a 4-dimensional matrix, however, it must be a 3-dimensional one. The procedure for changing 4-dimensional assignment matrix into the 3-dimensional one is as follows:

```
Begin
    Let yy=zeros (I, L, J);
    Let j=1;
      While j<w do
      begin
         E=zeros (I, L, 1);
         Let h=1;
            While h<H do
            begin
               Let E=E+Y (:,:,j,h);
               Let h=h+1;
            end
         Let yy (:,:,j)=E;
         Let j=j+1;
      end
    Let Y=yy;
  End.
```

Now, the procedure for solving the third sub-problem is finished and the values of X_{jh} and Y_{ijl} $(\forall i, j, l)$ for the relaxed model (Z_{LR}) are available.

4.3. Finding an upper bound and a feasible solution for the primal problem

At each iteration of the Lagrangian procedure, we find an upper bound as follows. We initially fix the warehouse locations at those sites, for which $X_{jh} = 1$ in the current Lagrangian solution and $\overline{X}_{jh} = X_{jh}$, $\forall j = 1,...,J$, $\forall h = 1,...,H$.It is possible for the Y_{ijl} variable not to be feasible for the primal problem. In order to satisfy the case, we compute $d_{il} \cdot s_l$ for each retailer *i* and product *l*, then sort them in a decreasing order. Then we assign the sorted demands to warehouses in three cases. First, for each retailer i and product l if, $0 < \sum_j Y_{ijl} \leq 1$, then we put $Y_{ijl} = 1$. The second case occurs, when $\sum_j Y_{ijl} = 0$ (the demand of retailer *i* for product *l* is not assigned to any opened warehouse in the Lagrangian solution), in which we have to compute the cost of retailers' assignment and products to any opened warehouse and demands are assigned to the warehouses, considering the assignment cost and warehouse capacity. Finally, we process retailer i and product l, in which $\sum_j Y_{ijl} > 1$ (the demand of retailer i for product l is assigned to at least two opened warehouses in the Lagrangian solution) with the same procedure similar to the second one.

4.4. Improvement Mechanism

In this mechanism, we try to improve the calculated upper bound in the previous Section. First, consider all current retailers assignment to the opened warehouses and reassign them to the other ones if their unused capacity is enough. Second, calculate the allocation objective function for all new assignments. The values of new assignments are generally equal to the sum of the changes in the second, third and fourth terms of Eq. (12). However, if the reassigning product l of retailer i from a warehouse at site j to another warehouse will remove all of the assigned demands at site j, then the value of the move is augmented by the fixed cost, F_{jh}, of that warehouse (since we can remove the site).

4.5. Warehouse Exchange Mechanism

After finding a feasible solution for the primal problem, we apply a variant of exchange algorithm proposed by Teits and Bart (1968) for P-median problem. For each warehouse in the current solution in the selected capacity level, we find the best substitute warehouse in the same capacity level that is not in the current solution and has sufficient capacity to satisfy the demands assigned to the initial warehouse. For such a potential exchange, demands are assigned in a greedy manner to the warehouse which has minimum increase in the cost based on the assignments made so far. If a warehouse exchange is found to improves the solution and satisfy the capacity constraint, we make the exchange; otherwise, we proceed to the next opened warehouse and try to find an improving exchange involving that warehouse. If any improving exchange is found, we apply improvement mechanism (as discussed in Section 4.4) to the best warehouse configuration we have found and then restart the search for improving the exchanges. If a pass through all possible exchanges is made without finding an improving exchange, the exchange algorithm terminates. The procedure of this mechanism is provided in Appendix A.

4.6. Capacity Planning Mechanism

After applying improvement and exchange mechanisms on feasible solution, for further decreasing the fix cost for opening and operating warehouses, the capacity planning mechanism may be considered. For all warehouses whose location variable is one ($X_{jh} = 1$), this condition must be checked: 'Decreasing warehouse capacity level is possible or not?' If the answer is yes, then the warehouse capacity will be decreased to the next lower level. This procedure will be stopped when the answer of the condition changes to No.

4.7. Adjusting the Lagrangian multipliers

The process of adjusting the Lagrangian multipliers requires the calculation of the direction of movement and the step size. In this paper, we use the sub-gradient method, which consider violation-matrixes as the ascending direction. The step size at the k^{th} iteration can be written as follows:

$$\omega^k = \rho^k \frac{(Z_k^{Sup} - Z_k^{Inf})}{\left\|VD^k\right\|^2 + \left\|VV^k\right\|^2 + \left\|VS^k\right\|^2} \qquad (39)$$

where Z_k^{Sup} is a value of the best (smallest) feasible solution found so far, and Z_k^{Inf} is a value of the solution to the Lagrangian model (*LR*) at the current iteration (*k*). Furthermore, ρ^k is a parameter between 0 and 2. The other components are:

$$VD_{jl}^{k} = \sum_{i=1}^{I} Y_{ijl}^{k} \cdot d_{il} - D_{jl}^{k} \qquad \forall j = 1,...,J,\ \forall l = 1,...,L$$

$$VV_{jl}^{k} = \sum_{i=1}^{I} Y_{ijl}^{k} \cdot v_{il} - V_{jl}^{k} \qquad \forall j = 1,...,J,\ \forall l = 1,...,L \tag{40}$$

$$VS_{il}^{k} = \sum_{j=1}^{J} Y_{ijl}^{k} - 1 \qquad \forall i = 1,...,I,\ \forall l = 1,...,L$$

Thus, the updating of the Lagrangian multipliers are as follows:

$$\alpha_{il}^{k+1} = \alpha_{il}^{k} - \omega^{k} \cdot VS_{il}^{k}$$

$$\beta_{jl}^{k+1} = Max\left\{0, \beta_{jl}^{k} + \omega^{k} \cdot VD_{jl}^{k}\right\} \tag{41}$$

$$\gamma_{jl}^{k+1} = Max\left\{0, \gamma_{jl}^{k} + \omega^{k} \cdot VV_{jl}^{k}\right\}$$

The procedure to obtain Z_{k}^{Sup} consists, in to calculating a feasible solution for the primal problem, which in this case, is based on the solution stated for $SP3^{k}$, considering:

$$\overline{D}_{jl}^{k} = \sum_{i=1}^{I} Y_{ijl}^{k} \cdot d_{il} \quad , \quad \overline{V}_{jl}^{k} = \sum_{i=1}^{I} Y_{ijl}^{k} \cdot v_{il} \tag{42}$$

Furthermore,

$$Z_{k}^{Sup} = Min\left\{Z_{k-1}^{Sup}, \overline{Z}_{k}\right\} \quad , \quad Z_{k}^{Inf} = Max\left\{Z_{k-1}^{Inf}, \underline{Z}_{k}\right\} \tag{43}$$

Where

$$\overline{Z}_{k} = \sum_{j=1}^{J}\sum_{h=1}^{H} F_{jh} \cdot X_{jh}^{k} + \sum_{j=1}^{J}\sum_{l=1}^{L} CH_{jl} \cdot \sqrt{\overline{D}_{jl}^{k}} + \sum_{j=1}^{J}\sum_{l=1}^{L} CS_{jl} \cdot \sqrt{\overline{V}_{jl}^{k}} + \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L} CT_{ijl} \cdot Y_{ijl}^{k} \tag{44}$$

Finally, $\underline{Z}_{k}$ is calculated as follows:

$$\underline{Z}_{k} = \sum_{j=1}^{J}\sum_{h=1}^{H} F_{jh} \cdot X_{jh}^{k} + \sum_{j=1}^{J}\sum_{l=1}^{L} (CH_{jl} \cdot \sqrt{D_{jl}^{k}} - \beta_{jl}^{k} \cdot D_{jl}^{k}) + \sum_{j=1}^{J}\sum_{l=1}^{L} (CS_{jl} \cdot \sqrt{V_{jl}^{k}} - \gamma_{jl}^{k} \cdot V_{jl}^{k})$$

$$+ \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L} (CT_{ijl} + \beta_{jl}^{k} \cdot d_{il} + \gamma_{jl}^{k} \cdot v_{jl} - \alpha_{il}^{k}) \cdot Y_{ijl}^{k} + \sum_{i=1}^{I}\sum_{l=1}^{L} \alpha_{il}^{k} \tag{45}$$

The algorithm is stopped when some convergence conditions are met (see Appendix B).

5. Computational Results

The computational experiments described in this section were designed to evaluate the performance of the proposed solution procedure with respect to a series of test problems. Since there is no benchmark model for the MDNDMC presented here for the small-size problems, the total cost obtained from LR is compared with the total cost resulting from LINGO 8.00 optimization software. Moreover, some large-size problems which cannot be solved by LINGO or other commercial softwares are only solved by the proposed LR. Comparison between the results of the LR and LINGO, for small-size problems, shows that we can also trust the LR for the larger problem sizes.

5.1. Designing Test Problems

Various test problems, with different sizes, are solved to evaluate the performance of the presented algorithm. The sizes of the test problems considered by some researchers, and the sizes of the designed test problems are listed in Tables 2 and 3, respectively. For each problem size, a series of problems are designed with different combinations of the parameters' values, in order to simulate different situations of real-world cases.

Fiftheen problem sets were generated randomly but systematically to capture a wide range of problem structures. Ten problems from each group with the same structure were solved in order to achieve a reasonable level of confidence about the performance of the solution procedure on that problem structure. A total of 150 problem instances were solved. The numbers of customers and potential warehouse sites vary from 40 to 150 and from 10 to 30, respectively. The number of products was fixed to 2, 3 or 5.

The required parameters for these problems are extracted from the following uniform distributions:

$$TC_{ijl} \approx U(0,200),\ \ \overline{TC}_{jl} \approx U(0,200),\ \ s_l \approx U(0,200), PH=1000, K=1.96$$

Five capacity levels were used for the capacities available to the potential warehouses (i.e., H=5).

Table 2. Some test problems' size in literature

References	No. of products	No. of suppliers	No. of warehouses	No. of retailers
Qu et al. (1999)	15-20	7	1	-
Sabri and Beamon (2000)	2	3	4	5
Jayaraman and Pirkul (2001)	10	5	15	75
Hwang (2002)	1	-	4	50-99
Syam (2002)	5	100	20	-
Syarif et al. (2002)	1	6-15	8-12	50-100
Zhou et al. (2002)	-	-	10	100
Jayaraman and Ross (2003)	2-3	5	10-15	30-75
Chen and Lee (2004)	2	-	2	2
Miranda and Garrido (2004)	1	-	10	20
Wang et al. (2004)	2	-	2	2
Melachrinoudis et al. (2005)	1	-	21	281
Altiparmak et al. (2006)	1	5	6	63
Amiri (2006)	1	-	10-25	100-500
Fearahani and Elahipanah (2008)	2-8	2-8	2-15	4-60
Miranda and Garrido (2008)	1	-	20	40

Table 3. The structure of test problems

Problem set	No. of customers	No. of products	No. of potential warehouses	No. of constraints	Problem size	
					No. of integer variables	No. of nonlinear variables
1	40	2	10	990	850	40
2	40	3	10	1450	1250	60
3	40	5	10	2370	2050	100
4	50	2	15	1765	1575	60
5	50	3	15	2595	2325	90
6	50	5	15	4255	3825	150
7	75	2	20	3370	3100	80
8	75	3	20	4985	4600	120
9	75	5	20	8215	7600	200
10	100	2	20	4420	4100	80
11	100	3	20	6560	6100	120
12	100	5	20	10840	10100	200
13	150	2	20	6520	6100	80
14	150	3	20	9710	9100	120
15	150	5	30	23760	22650	300

5.2. Discussion

The Lagrangian heuristic is coded in Matlab 7, and LINGO 8.0 software is used to compare the results of the problem sets. All the test problems are solved on a Pentium 4 computer with

448MB RAM and 2.0GHz CPU. Table 4 summarizes the numerical results of the proposed solution procedures. The results are described by providing the number of improvements (out of 10) that LR's solution were better than LINGO solution, the average percentage decrease in the cost of LR's solution compared to the LINGO solution (% save). The gap, defined as $100\times$(improved the upper bound-lower bound)/lower bound, is used to evaluate the quality of the solutions. We also include the average warehouse capacity utilization in Table 4. Finally, we report the average and the worst CPU times in seconds for the Lagrangian heuristics and LINGO.

Hence, we considered the best-found after running the corresponding LINGO models for 180 minutes. As it is indicated in Table 4 for the problem sets 9 and 11-15, even no feasible solution is found after running the corresponding LINGO model for 180 minutes (These cases are indicated by (-) in Table 4). For this reason, we do not compare LINGO results with the proposed Lagrangian heuristic in these problems. The results of the reported experiments in Table 4, show that the proposed LR heuristic produces very good feasible solutions compared to the results generated from LINGO in significantly less CPU time. The average of CPU time for LINGO results is between 1066 to 8132 seconds (except problem sets 9 and 11-15), but this time for LR heuristic is between 35.1 to 233 seconds.

Table 4 shows that the proposed solution approach does well for a wide range of problem sizes, with a mean average gap of 0.89% (from 0.51% to 1.58%) over all 150 test problems. The number of customers does not seem to have a significant effect on the quality of the solution procedure (especially in large scale). It seems, however, that for test problems with a given number of customer zones, the gap for problems with small number of potential warehouse sites is smaller than others.

In all test problems used in Table 4, the capacities of warehouses are efficiently utilized as in[illegible]ion measures.

Table 4. Computational results

Problem set	No. of Imp.	Save (%)	Average of W.L.R. (%)	GAP (%)		CPU time for MDNDMC-LINGO		CPU time for MDNDMC- LR	
				Average	Worst	Average	Worst	Average	Worst
1	7	2.35	92.3	0.51	0.88	1066	1939	35.1	39
2	6	2.71	93.8	0.57	0.79	1451	2526	53.5	67
3	8	3.72	94.1	0.62	0.91	2142	3453	59.2	78
4	6	2.88	92.3	0.59	0.83	2011	2838	55.1	74
5	7	4.73	91.1	0.71	0.97	4480	6211	64	83
6	9	5.12	93.2	0.78	1.04	8132	10800	79.1	91
7	10	6.25	91.4	0.91	0.95	7951	9625	77	87
8	10	6.11	93.2	0.87	0.94	5101	8983	55	94
9	-	-	94.2	1.02	1.09	-	-	87.6	105
10	10	6.32	91.3	0.90	0.97	6140	8764	81	97
11	-	-	93.1	0.94	1.04	-	-	91	102
12	-	-	92.3	0.92	1.11	-	-	95.2	121
13	-	-	89.4	1.16	3.30	-	-	132	611
14	-	-	94.5	1.28	2.31	-	-	203	413
15	-	-	90.0	1.58	2.00	-	-	233	678

Warehouse load ratio (W.L.R.) = total demand of retailer/ total capacity of selective warehouses.
GAP(%)= [(improved upper bound –lower bound)/ lower bound] $\times$ 100.
No. Imp.= No. of improvements (out of 10) that LR's results were better than LINGO results.

For more evaluation, the effect of warehouse capacity flexibility in capacity utilization, the proposed model has been formulated with fix warehouse capacity. Table 5 shows the average warehouse utilization obtained from MDNDMC and multi-product distribution network design model with fix warehouse capacity (MDNDFC).

The mean average percentage of warehouse load ratio (W.L.R.) for MDNDMC and MDNDFC are 92.7% and 88.3%, respectively. In other words, if the distribution network problem formulated with flexible warehouse capacity level, capacity planning would improved warehouse load ratio. This improvement is caused by capacity planning mechanism in proposed LR heuristic.

Table 5. Average of warehouse utilization (%) from MDNDMC and MDNDFC

Problem set	1	2	3	4	5	6	7	8	9	10	11	12	Average
MDNDMC	92.3	93.8	94.1	92.3	91.1	93.2	91.4	93.2	94.2	91.3	93.1	92.3	92.7
MDNDFC	87.3	89.8	90.1	88.3	90.3	85.2	87.4	91.2	85.2	88.8	85	91	88.3

6. Concluding Remarks

In this paper, an attempt is made to integrate location, allocation, and inventory decisions in the design of a supply chain distribution network. The model was formulated as a mixed integer non-linear programming problem and was solved by using Lagrange Relaxation with a sub-gradient search method. The model was decomposed into the location/allocation module and inventory module. The results for the randomly selected problems show that the gap in an objective function value ranges is between 0.51 and 1.58 percent. In addition, from the CPU time point of view, the performance was very good (35.1 up to 233 seconds).

Unlike most of the past research, our study allows for the multiple levels of capacities available for the warehouses in a multi-product system. The computational results show a good effect of capacity flexibility in warehouse capacity utilization. The results of extensive computational tests indicate that the procedure is both effective and efficient for a wide variety of problem sizes and structures. The proposed model in this paper not only minimizes the total distribution and inventory costs but also considers the customer service level and warehouse capacity utilization.

In terms of future research, it would be interesting to apply this simultaneous methodology to more complex supply chains with multi-type, multi-level warehouses. For better customer service level, it is possible to introduce some opened warehouses as the main supplier warehouses, multi-type warehouses, to satisfy the other warehouses in their lead time. In some real-world situations, for backorder avoiding, some warehouses must be supported by others. Recognition of the main warehouses and allocation of them to other ones must be formulated. Furthermore, it is possible to consider different levels of shared information between the plants and warehouses, to modeling the bullwhip effect and its impact on the supply chain distribution network design.

Appendix A

In this appendix we show the procedure for exchange mechanism that introduced in the Section 4.5.

Warehouse exchange procedure:

```
Begin
    Let e=1;
    Let k=1;
    while (e<size (E)) do
    begin
      while (k<size(K)) do
      begin
        if [(cost (k, h)<cost (e, h)) && ( câp_kh < cap_eh )]
        let X̄_eh = 0;
        let X̄_kh = 1;
        let dd= Y (:,:,e);
        let Y (:,:,k)=Y (:,:,k)+ dd;
        let câp_kh = câp_kh − Σ_{i=1}^{I} Σ_{l=1}^{L} s_l · d_il · Y(i,l,e);
        else
        let k=k+1;
      end
    end
   e=e+1;
End.
```

E is a vector of the current solution for the located warehouses which location variable in each capacity level is one ($\overline{X}_{eh} = 1,\ \forall e \in E,\ \forall h \in H$). K is a vector of the warehouses that in current solution its location variable is zero ($\overline{X}_{kh} = 0,\ \forall k \in K,\ \forall h \in H$). $\overline{X}_{jh}$ is the location variable for site j and cap_{jh} is the capacity of warehouse on site j with capacity level h. Furthermore, $\hat{cap}_{jh}$ is the available capacity of warehouse on site j with capacity level, h, at each iteration.

Appendix B

In this appendix, we show the generic version of the LR heuristic used to solve the MDNDMC model. LR heuristic (with proposed procedures) was implemented in Matlab 7.

Generic scheme procedure of the LR heuristic for MDNDMC:

Begin
    Initialize the Lagrangian multipliers ($\alpha_{il}, \beta_{jl}, \gamma_{jl}$ *) for every*
    $i = 1,...,I$, $j = 1,...,J$, $l = 1,...,L$ *;*
    let k=1;
    let p=0;
    while *(* $k \le N\max 1$ *) and* $(p \le N\max 2)$ ***do***
    begin
        solve the sub-problems $SP1^k$ *,* $SP2^k$ *and* $SP3^k$ *(based on the procedure stated in sub-section 4.2);*
        compute $\overline{X}_{jh}, Y_{ijl}, \overline{D}_{jl}$ *and* $\overline{V}_{jl}$ *for every*
        $i = 1,...,I$, $j = 1,...,J$, $l = 1,...,L$, $h = 1,...,H$ *;*
        run the improvement and warehouse exchange procedures (based on the mechanisms stated in sub-sections 4.4 and 4.5);
        run the capacity planning procedure (based on the mechanism stated in sub-section 4.6);
        compute violations for every relaxed constraints , VS^k *,* VD^k *and* VV^k *, based on Eq. (40);*
        compute lower and upper bound for every iteration k, Z_k^{Inf} *,* Z_k^{Sup} *, respectively, using Eq. (43);*
        update Lagragian multipliers, α_{il}, β_{jl} *and* γ_{jl} *for*
        $i = 1,...,I$, $j = 1,...,J$, $l = 1,...,L$ *, according to sub-section (4.7);*
        update counters k and p;
        if (%duality gap $\le$ *0.1)*
        break;
    end
End.

k is the iteration counter of the algorithm, which has a maximum of *N max1* and p counts the consecutive iteration in which the upper bound has not been improved, for which there is a limit given for *N max2*.

References

Altiparmak, F., Gen, M., Lin, L. and Paksoy, T. (2006), "A genetic algorithm approach for multi-objective optimization of supply chain networks", *Computers & Industrial Engineering*, Vol. 51, pp. 197-216.

Amiri, A. (2006), "Designing a distribution network in a supply chain system: formulation and efficient solution procedure", *European Journal of Operational Research*, Vol. 171, pp. 567-576.

Brown, G.G., Graves, G.W. and Honczarenko, M.D. (1987), "Design and operation of a multicommodity production/distribution system using primal goal decomposition", *Management Science*, Vol. 33, pp. 1469-1480.

Chan, F.T.S., Chung, S.H. and Wadhwa, S. (2005), "A hybrid genetic algorithm for production and distribution", *Omega*, Vol. 33, pp. 345-355.

Chen, C.L. and Lee, W.C. (2004), "Multi-objective optimization of multi-echelon supply chain networks with uncertain product demands and prices", *Computers & Chemical Engineering*, Vol. 28, pp. 1131–1144.

Chung, S.-h., Myung, Y.-S. and Tcha, D.-W. (1992), "Optimal design of a distributed network with a two-level hierarchical structure", *European Journal of Operational Research*, Vol. 62 No. 1, pp. 105–115.

Cohen, M.A. and Moon, S. (1990), "Impact of Production Scale Economies, Manufacturing Complexity and Transportation Costs on Supply Chain Facility Networks", *Journal of Manufacturing and Operations Management*, Vol. 3, pp. 269-292.

Dantzig, G.B. (1957), "Discrete variables extremum problems", *Operations Research*, Vol. 5, pp. 266–277.

Daskin, M.S., Coullard, C.R. and Shen, Z-J.M. (2002), “An Inventory-Location Model: Formulation, Solution Algorithm and Computational Results”, *Annals of Operations Research*, Vol. 110, pp. 83–106.

Drezner, Z., Klamroth, K., Schobel, A., and Wesolowsky, G. (2002), “In Z. Drezner & Hamacher (Eds.), *The Weber problem in facility location: applications and theory*.” Berlin: Springer.

Eksioglu, S.D., Romeijn, H.E. and Pardalos, P.M. (2006), Cross-facility management of production and transportation planning problem. *Computers & Operations Research*, Vol. 33, pp. 3231- 3251.

Eskigun, E., Uzsoy, R., Preckel, P.V., Beaujon, G., Krishnan, S. and Tew, J.D. (2005), “Outbound supply chain network design with mode selection, lead times and capacitated vehicle distribution centers”, *European Journal of Operations Research*, Vol. 165, pp. 182-206.

Farahani, R.Z. and Elahipanah, M. (2008), “A genetic algorithm to optimize the total cost and service level for just-in-time distribution in a supply chain”, *International Journal of Production Economics*, Vol. 111, pp. 229-243.

Gourdin, E., Labbe, M. and Laporte, G. (2000), “The uncapacitated facility location problem with client matching”, *Operations Research*, Vol. 48, pp. 671–685.

Hurter, A.P. and Martinich, J.S. (1989), *Facility location and the theory of production*. Kluwer Academic Publishers Group, Boston, Mass.

Hwang, H.S. (2002), “Design of supply-chain logistics system considering service level”, *Computers & Industrial Engineering*, Vol. 43, pp. 283-297.

Jayaraman, V., (1998), “An efficient heuristic procedure for practical sized capacitated warehouse design and management”, *Decision Sciences*, Vol. 29, pp. 729-745.

Jayaraman, V. and Pirkul, H. (2001), “Planning and coordination of production and distribution facilities for multiple commodities”, *European Journal of Operations Research*, Vol. 133, pp. 394-408.

Jayaraman, V. and Ross, A. (2003), “A simulated annealing methodology to distribution network design and management”, *European Journal of Operations Research*, Vol. 144, pp. 629-645.

Liang, T.F. (2006), “Distribution planning decisions using interactive fuzzy multi-objective linear programming”, *Fuzzy Sets and Systems*, Vol. 46, pp. 1303-1316.

Melachrinoudis, E., Messac, A. and Min, H. (2005), “Consolidating a warehouse network: a physical programming approach”, *International Journal of Production Economics*, Vol. 97, pp. 1-17.

Melachrinoudis, E. and Min, H. (2000), “The dynamic relocation and phase-out of a hybrid, two-echelon plant/warehousing facility: A multiple objective approach”, *European Journal of Operations Research*, Vol. 123, pp. 1-15.

Miranda, P.A. and Garrido, R.A. (2004), “Incorporating inventory control decisions into a strategic distribution network design model with stochastic demand”, *Transportation Research Part E*, Vol. 40, pp. 183–207.

Miranda, P.A. and Garrido, R.A. (2008), “Valid inequalities for Lagrangian relaxation in an inventory location problem with stochastic capacity”, *Transportation Research Part E*, Vol. 44, pp. 47-65.

Nagar, L. and Jain, K. (2008), “Supply chain planning using multi-stage stochastic programming”, *Supply Chain Management: An International Journal*, Vol. 13, No. 3, pp. 251-256.

Nonino, F. and Panizzolo, R. (2007), “Integrated production/distribution planning in the supply chain: the Febal case study ”, *Supply Chain Management: An International Journal*, Vol. 12, No. 2, pp. 150-163.

Nozick, L.K. (2001), “The fixed charge facility location problem with coverage restrictions”, *Transportation Research Part E*, Vol. 37, pp. 281-296.

Pirkul, H. and Jayaraman, V. (1996), “Production, Transportation and Distribution Planning in a Multi-Commodity Tri-Echelon System”, *Transportation Science*, Vol. 30 No. 4, pp. 291-302.

Pirkul, H. and Jayaraman, V. (1998), “A Multi-Commodity, Multi-Plant, Capacitated Facility Location Problem: Formulation and Efficient Heuristic Solution”, *Computers & Operations Research*, Vol. 25 No. 10, pp. 869-878.

Qu, W.W., Bookbinder, J.H. and Iyogun, P. (1999), “An integrated inventory- transportation system with modified periodic policy for multiple products”, *European Journal of Operational Research*, Vol. 115, pp. 254-269.

Sabri, E.H. and Beamon, B.M. (2000), “A multi-objective approach to simultaneous strategic and operational planning in supply chain design”, *Omega*, Vol. 28, pp. 581-598.

Shen, Z.-J.M., Coullard, C. and Daskin, M.S. (2003), “A joint location–inventory model”, *Transportation Science*, Vol. 37 No. 1, pp. 40–55.

Syam, S.S. (2002), “A model and methodologies for the location problem with logistical components”, *Computers & Operations Research*, Vol. 29, pp. 1173-1193.

Syarif, A., Yun, Y.S. and Gen, M. (2002), “Study on multi-stage logistic chain network: a spanning tree-based genetic algorithm approach”, *Computers & Industrial Engineering*, Vol. 43, pp. 299-314.

Teits, M.B. and Bart, P. (1968), “Heuristic methods for estimating the generalized vertex m[illegible]*erations Research*, Vol. 16 No.5, pp. 955-961.

Tragantalerngsak, S., Holt, J. and Ronnqvist, M. (2000), "An exact method for the two-echelon, single-source, capacitated facility location problem", *European Journal of Operational Research*, Vol. 123, pp. 473–489.

Wang, W., Fung, R.Y.K. and Chai, Y. (2004), "Approach of just-in-time distribution requirements planning for supply chain management", *International Journal of Production Economics*, Vol. 91, pp. 101-107.

Zhou, G., Min, H. and Gen, M. (2002), "The balanced allocation of customers to multiple distribution centers in supply chain network: a genetic algorithm approach", *Computers & Industrial Engineering*, Vol. 43, pp. 251-261.

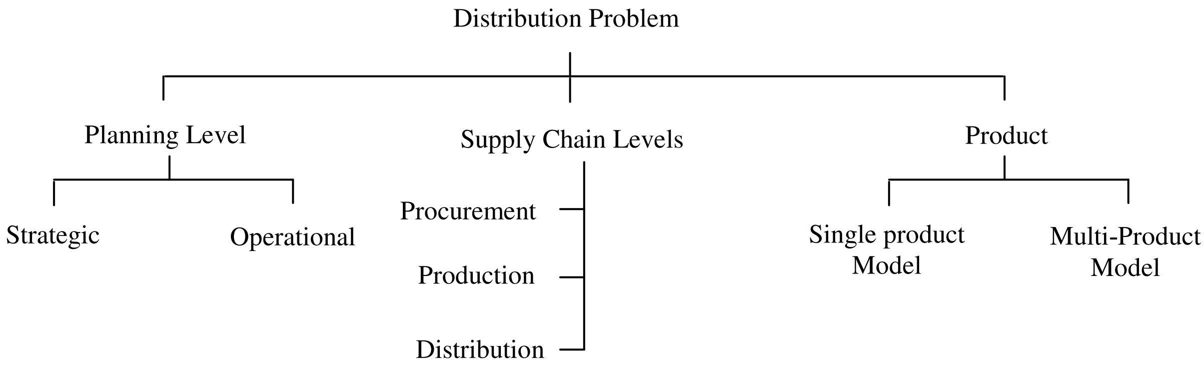


Fig 1. Classification of Distribution Problems in Supply Chain Management

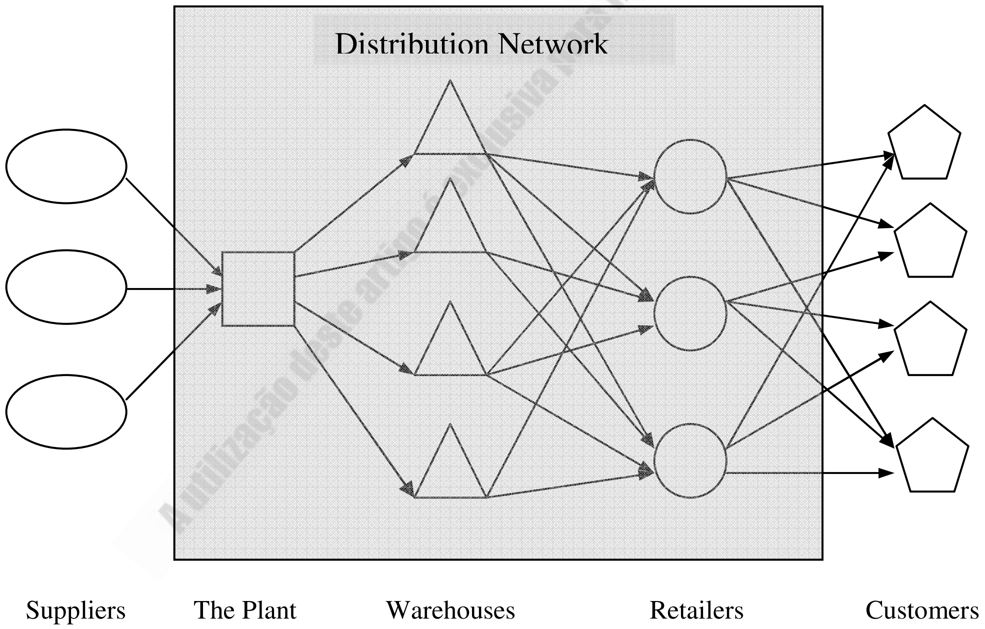


Fig 2. Stricture of the supply chain distribution network

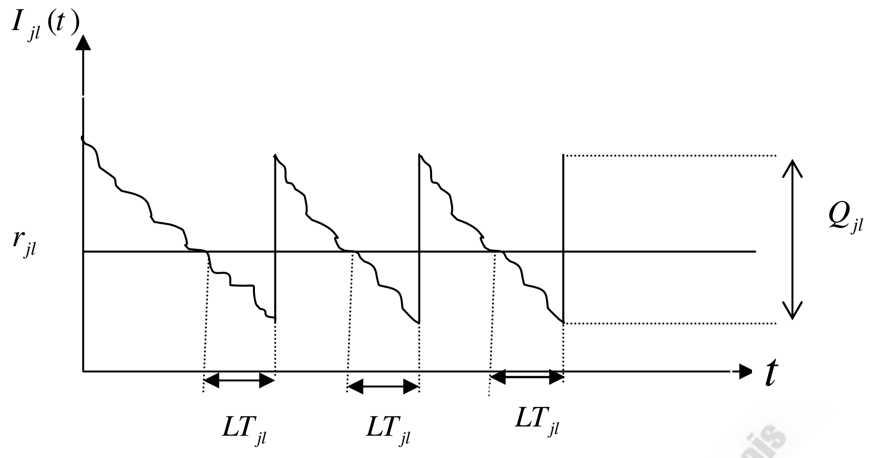


Fig. 3. Evolution of the inventory level $I_{jl}(t)$ at site j

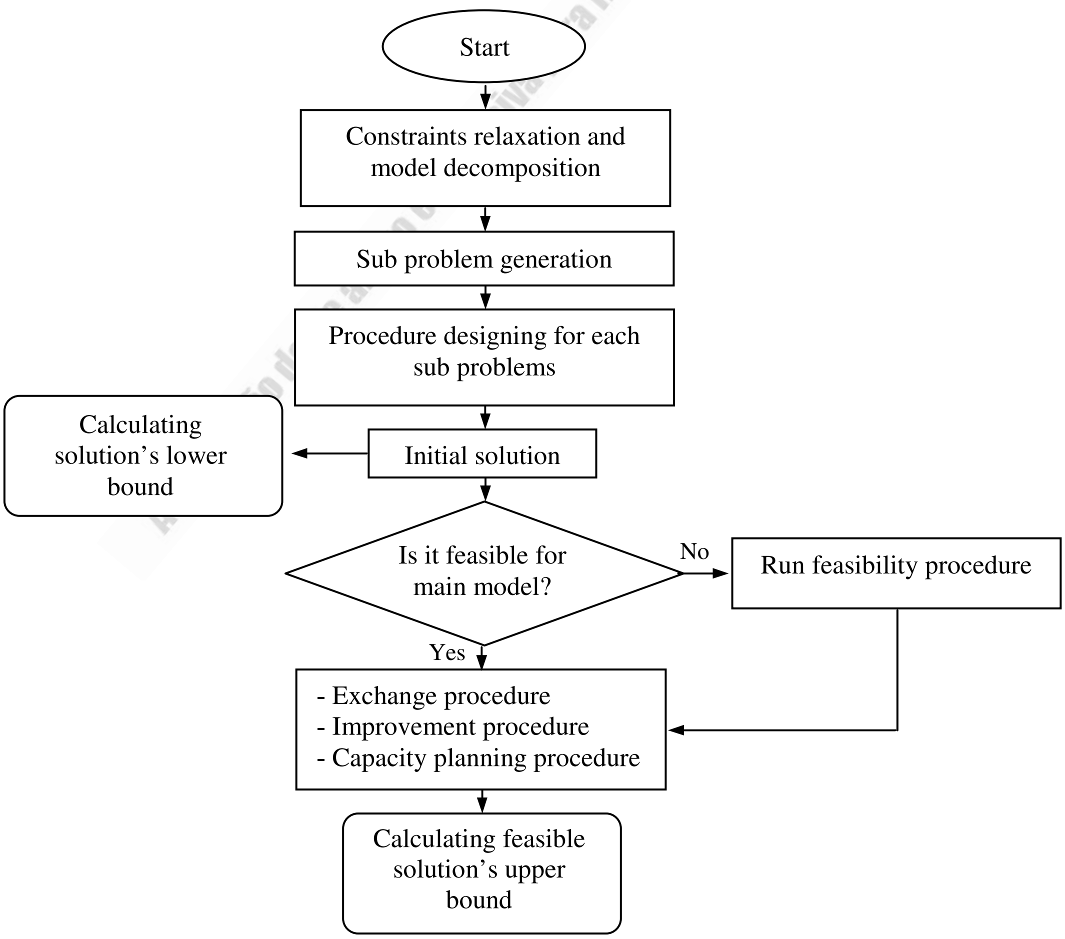


Fig. 4. The framework of the proposed LR algorithm

Table 1. Classification of Distribution Problems

Planning Level	Strategic	Brown et al. (1987), Cohen and Moon (1990), Pirkul and Jayaraman (1996, 1998), Melachrinoudis and Min (2000), Zhou et al. (2002), Eskigun et al. (2005), Altiparmak et al. (2006), Amiri (2006)
	Operational	Que et al. (1999), Wang et al. (2004), Chan et al. (2005)
	Strategic/ Operational	Sabri and Beamon (2000), Jayaraman and Pirkul (2001), Nozick (2001), Hwang (2002), Syarif et al. (2002), Jayaraman and Ross (2003), Chen and Lee (2004), Miranda and Garrido (2004), Melachrinoudis et al. (2005), Miranda and Garrido (2008)
Supply Chain Levels	Procurement	
	Production	Wang et al. (2004), Chan et al. (2005)
	Distribution	Brown et al. (1987), Pirkul and Jayaraman (1998), Nozick (2001), Zhou et al. (2002), Eskigun et al. (2005), Amiri (2006)
	Production/ Distribution	Cohen and Moon (1990), Pirkul and Jayaraman (1996), Melachrinoudis and Min (2000), Hwang (2002), Jayaraman and Ross (2003), Chen and Lee (2004), Miranda and Garrido (2004), Melachrinoudis et al. (2005), Nonino and Panizzolo (2007), Miranda and Garrido (2008)
	Procurement/ Distribution	Que et al. (1999)
	Procurement/ Production/ Distribution	Sabri and Beamon (2000), Jayaraman and Pirkul (2001), Syarif et al. (2002), Altiparmak et al. (2006), , Nagar and Jain (2008)
Product	Single product model	Nozick (2001), Hwang (2002), Syarif et al. (2002), Miranda and Garrido (2004), Chan et al. (2005), Melachrinoudis et al. (2005), Altiparmak et al. (2006), Amiri (2006), Miranda and Garrido (2008)
	Multi-product model	Brown et al. (1987), Cohen and Moon (1990), Pirkul and Jayaraman (1996, 1998), Que et al. (1999), Melachrinoudis and Min (2000), Sabri and Beamon (2000), Jayaraman and Pirkul (2001), Jayaraman and Ross (2003), Chen and Lee (2004), Wang et al. (2004), Eskigun et al. (2005)

Table 2. Some test problems' size in literature

References	No. of products	No. of suppliers	No. of warehouses	No. of retailers
Altiparmak et al. [1]	1	5	6	63
Amiri [2]	1	-	10-25	100-500
Chen and Lee [5]	2	-	2	2
Fearahani and Elahipanah [12]	2-8	2-8	2-15	4-60
Hwang [15]	1	-	4	50-99
Jayaraman and Pirkul [17]	10	5	15	75
Jayaraman and Ross [18]	2-3	5	10-15	30-75
Melachrinoudis et al. [20]	1	-	21	281
Miranda and Garrido [22]	1	-	10	20
Miranda and Garrido [23]	1	-	20	40
Qu et al. [27]	15-20	7	1	-
Sabri and Beamon [28]	2	3	4	5
Syam [30]	5	100	20	-
Syarif et al. [31]	1	6-15	8-12	50-100
Wang et al. [34]	2	-	2	2
[illegible]	-	-	10	100

Table 3. The structure of test problems

Problem set	No. of customers	No. of Products	No. of potential warehouses	No. of constraints	Problem size	
					No. of integer variables	No. of nonlinear variables
1	40	2	10	990	850	40
2	40	3	10	1450	1250	60
3	40	5	10	2370	2050	100
4	50	2	15	1765	1575	60
5	50	3	15	2595	2325	90
6	50	5	15	4255	3825	150
7	75	2	20	3370	3100	80
8	75	3	20	4985	4600	120
9	75	5	20	8215	7600	200
10	100	2	20	4420	4100	80
11	100	3	20	6560	6100	120
12	100	5	20	10840	10100	200
13	150	2	20	6520	6100	80
14	150	3	20	9710	9100	120
15	150	5	30	23760	22650	300

Table 4. Computational results

Problem set	No. of Imp.	Save (%)	Average of W.L.R. (%)	GAP (%)		CPU time for MDNDMC-LINGO		CPU time for MDNDMC- LR	
				Average	Worst	Average	Worst	Average	Worst
1	7	2.35	92.3	0.51	0.88	1066	1939	35.1	39
2	6	2.71	93.8	0.57	0.79	1451	2526	53.5	67
3	8	3.72	94.1	0.62	0.91	2142	3453	59.2	78
4	6	2.88	92.3	0.59	0.83	2011	2838	55.1	74
5	7	4.73	91.1	0.71	0.97	4480	6211	64	83
6	9	5.12	93.2	0.78	1.04	8132	10800	79.1	91
7	10	6.25	91.4	0.91	0.95	7951	9625	77	87
8	10	6.11	93.2	0.87	0.94	5101	8983	55	94
9	-	-	94.2	1.02	1.09	-	-	87.6	105
10	10	6.32	91.3	0.90	0.97	6140	8764	81	97
11	-	-	93.1	0.94	1.04	-	-	91	102
12	-	-	92.3	0.92	1.11	-	-	95.2	121
13	-	-	89.4	1.16	3.30	-	-	132	611
14	-	-	94.5	1.28	2.31	-	-	203	413
15	-	-	90.0	1.58	2.00	-	-	233	678

Warehouse load ratio (W.L.R.) = total demand of retailer/ total capacity of selective warehouses.

GAP(%)= [(improved upper bound –lower bound)/ lower bound] $\times$ 100.

No. Imp.= No. of improvements (out of 10) that LR's results were better than LINGO results.

Table 5. Average of warehouse utilization (%) from MDNDMC and MDNDFC

Problem set	1	2	3	4	5	6	7	8	9	10	11	12
MDNDMC	92.3	93.8	94.1	92.3	91.1	93.2	91.4	93.2	94.2	91.3	93.1	92.3
MDNDFC	87.3	89.8	90.1	88.3	90.3	85.2	87.4	91.2	85.2	88.8	85	91