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'Heterogeneous enlargement', regional convergence and international competitiveness: a model-based analysis

ABSTRACT

The enlargement of a highly developed trading bloc or economic union by much less developed countries ('heterogeneous enlargement') and the subsequent convergence within the enlarged area presumably affect the international competitiveness of the latter vis-à-vis other large economies. The purpose of this article is to provide a model-based answer to the question that which are the impacts that the enlargement shock and following regional convergence have on the international competitiveness of the enlarged region measured by its terms of trade. Methodologically, the paper extends Zee's (1987) two-country model of the world economy by incorporating the enlargement of Home by a less-developed region. It is found that in the long run the terms of trade of enlarged Home deteriorate (Home's international competitiveness declines) by approximately that percentage by which the domestic population increases. As a consequence, Home's welfare decreases while Foreign's welfare increases in the long run. Along the transition path towards full convergence, the terms of trade of enlarged

KEYWORDS

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Home worsen and the capital intensity of the less-developed region in Home monotonically increases while the capital intensities of the developed Home region and of Foreign temporarily decrease.

1. INTRODUCTION

Recent decades have seen the enlargement of trading blocs and economic unions embracing both highly and less developed nations, a phenomenon aptly termed 'heterogeneous integration' (Wagner 2002) or 'heterogeneous enlargement'.[1] Heterogeneous enlargement might be seen as a major organizational transformation of the enlarged area which affects the economic convergence of the less towards higher developed regions as well as the international competitiveness of the enlarged area vis-à-vis other large economies. While leading politicians claim that enlargement will increase the international competitiveness of the enlarged area, economists are not able to demonstrate empirically that population-rich countries are more prosperous than smaller countries (Rose 2006). In politics the population size of a country apparently matters, but in both empirical and theoretical economics (in particular in neoclassical growth theory) it does not at all.

Thus, the question arises whether newer developments of economic growth theory leave room for the impacts of enlargement on intra-area convergence and international competitiveness or political wisdom cannot be related to economic reasoning. The purpose of this article is to show that the former is true by presenting a new growth theoretical general equilibrium model of the world economy in which the enlargement of an economic area through rise in the number of participants in its commodity and capital markets does have economic effects.

In particular, the article investigates the dynamic impacts of the increase in the population size of an economic union following from heterogeneous enlargement on the transition path of the world economy from a post-enlargement non-steady state to a full-convergence steady state. In contrast to neoclassical growth theory (Diamond 1965; Solow 1956), population size does affect the international competitiveness of the enlarged area in the modified neoclassical model of this article. As a measure of the international competitiveness the external terms of trade of the enlarged economic area are proposed. It will be shown that contrary to political rhetoric the terms of trade of the enlarged union deteriorate while capital accumulation (growth) of both the enlarged union and of another large open economy connected with the former through trade in commodities and financial capital temporarily decreases.

The related literature on the economic effects of (political) unification of economically heterogeneous countries or the economic impacts of the enlargement of economic unions by integrating less developed countries has used either closed-economy or small open-economy models. As regards the former, Funke and Strulik (2000) have studied the dynamic effects of interregional fiscal transfers on GDP growth and regional convergence of unified Germany. Included in the latter approach are macro-econometric and computable general equilibrium (CGE) models studying the welfare effects of the Eastern EU enlargement in 2004.[2] Neither the former nor the latter model approach is capable of eliciting the dynamic impacts of the enlargement between less-developed and developed region and related regional convergence on the

1 See the unification of Western and Eastern Germany and the Eastern enlargements of the European Union.

2 For an example of a macro-econometric model and a survey of related CGE models of EU enlargement, see Breuss (2002). An early study with regard to the welfare effects of the Eastern EU enlargement is Baldwin et al. (1997); more recent studies are provided by Heijdra et al. (2004) as well as Lejour et al. (2004).

international competitiveness of the enlarged economic area. Nor are they capable of investigating the impacts of the dynamics of the external terms of trade on capital accumulation (growth) of the enlarged area and on capital accumulation (growth) of the other, not enlarged country.

This article intends to examine these questions within a stylized model of the world economy, a two-country overlapping generations (OLG) model originally developed to study the international interdependence of fiscal policies of large open economies.[3] However, in such models, countries are assumed to be homogeneous and of equal, unchanging size. To depict the integration of market participants of less developed countries into the commodity and capital markets of the highly developed regions, the basic two-country OLG model is adapted by introducing regional disparities into one of the countries and by allowing for a change in its population size.

As is well known, the convergence of per-capita incomes of less developed countries towards those of high-income countries proceeds rather slowly,[4] while the equalization of regional rates of return on real capital within an economic region takes less time.[5] It is plausible to assume that within one period of a two-period OLG model (25–30 calendar years), real interest parity holds in the enlarged union. However, both capital market integration within one period and the more lengthy process of convergence cannot be introduced into Zee's (1987) two-country OLG model without modifications. Zee's model exhibits either real interest parity between the regions and immediate convergence, or real interest parity does not apply and convergence extends over several periods.

The solution of this specification problem consists of integrating Funke's and Strulik's (2000) two-region version of Barro's (1990) model of government spending and (endogenous) growth into Zee's (1987) exogenous-growth, two-country model. To avoid inconsistencies between Barro's scale model of endogenous growth and Zee's non-scale model of exogenous growth, Barro's original approach is modified in this article by assuming that private and public capitals do not exhibit constant but rather decreasing returns to scale.

This article contributes to the existing literature in several respects. First, it provides an economic-theoretical answer to the new question of how the enlargement of highly developed countries by less developed and the subsequent regional convergence influence the international competitiveness of the enlarged country measured by its external terms of trade. Second, Zee's two-country model is extended to a two-country, two-region OLG model with non-instantaneous convergence. Third, the transitional dynamics of the terms of trade and the private as well as public capital intensities after the enlargement shock is presented and economically interpreted.

The article is organized as follows. In the next section the main structure of the two-country, two-region model is presented, leaving the description of its inter-temporal equilibrium dynamics and the steady-state solution to the following section. In the fourth section the dynamic stability of steady-state solutions is briefly investigated and the long-run effects of Home's enlargement by a less-developed region are considered. In the fifth section the economic rationale behind the instantaneous jump of the terms of trade in response to the enlargement shock is provided. In the sixth section the convergence process after the enlargement and the dynamic transition of the terms of trade as well as of private and public capital intensities are described. Section 7 concludes.

3 See Buiter's (1981) seminal one-good and Zee's (1987) and Lin's (1994) two-good models.

4 Empirical estimations of the speed of convergence of Eastern European countries towards EU-15 can be found in Alho et al. (2005: 6). It is estimated that Eastern European member countries need at least 30–50 years to catch up with the old member countries.

5 Recent empirical investigations about the rates of return on investment in Eastern European countries report already shrinking rates of return less than two decades after their transition to market economies.

2. A TWO-COUNTRY, TWO-REGION OLG MODEL OF THE WORLD ECONOMY

The model presented in this article originates from the combination of two distinct modelling strands. The first consists of a log-linear, Cobb–Douglas version of Zee's (1987) original two-country model. The second relies on Funke and Strulik's (2000) endogenous growth model of a closed economy composed of two unequally developed regions.

The present model portrays a world economy with two interdependent countries, Home and Foreign. Home and Foreign are assumed to be identical with respect to consumer preferences and production technologies but different with respect to per-capita public debt levels. Moreover, one of the two countries, Home, is composed of two regions $r = R1, R2$, wherein $R2$ is the less and $R1$ is the more developed region. The two regions differ in terms of private capital intensities (per-capita incomes) and (per-capita) public infrastructure. Each country produces a specific composite commodity, which can be used for consumption as well as for public and private investment.[6] The domestically produced commodity is designated by x and the foreign-produced commodity by y^*.[7] Time is discrete.

6 In line with the macroeconomic nature of Zee's two-country model, the commodity-specialization pattern of each country remains open. It also implies that all commodities of each country are in principle tradable.

7 Henceforth, all variables referring to Foreign are denoted by an asterisk.

8 Freely flowing technological knowledge between both regions of Home as well as between Home and Foreign (see below) is assumed.

2.1. Firms

A large number of identical firms operate under perfect competition in both regions of Home as well as in Foreign. Their technology is specified according to a Cobb–Douglas production function. In region r of Home, private capital services K_t^r together with labour services N_t^r are employed to produce total output X_t^r under constant returns to scale:

$$X_t^r = A_t^r (K_t^r)^{\alpha} (N_t^r)^{1-\alpha} \qquad r = R1, R2 \tag{1}$$

As firms operate in a fully competitive environment, the production elasticity of capital (labour) services, $\alpha(1-\alpha)$ represents the capital (labour) income share.

A_t^r follows Barro's (1990) specification of the productivity effects of public infrastructure:

$$A_t^r = A_t \left(\frac{G_t^r}{L_t^r}\right)^{\eta} \qquad A_{t+1} = \Pi^A A_t, \quad \Pi^A > 0, \quad \eta < 1-\alpha \tag{2}$$

where A_t denotes total factor productivity, which changes by the exogenous growth factor of labour-saving technological progress Π^A and is equal for both regions.[8] G_t^r is the aggregate stock of infrastructure (public capital) of region r in period t. 'Infrastructure is a common external input to each firm's production function and is publicly provided' (Glomm and Ravikumar 1994: 1176). In order to avoid unwanted scale effects, it is assumed that the infrastructure stock per capita determines the total factor productivity of each firm (Funke and Strulik 2000: 366). As mentioned above, to make Barro's technological progress compatible with Zee's international OLG model with exogenous long-run growth rates, diminishing returns to scale to accumulating factors are assumed ($\eta < 1-\alpha$).

Maximization of real profits in region r, $r = R1, R2$, implies

$$q_t^r = \alpha A_t^r \left(\frac{K_t^r}{N_t^r}\right)^{\alpha-1} \tag{3}$$

$$w_t^r = (1-\alpha) A_t^r \left(\frac{K_t^r}{N_t^r}\right)^{\alpha} \tag{4}$$

As is well known, Equation (3) says that real capital service price (units of domestic good per capital unit) in region r is equal to the marginal product of capital in region r. Similarly, Equation (4) states that at a maximum profit, the real wage rate (units of domestic good per labour unit) in region r equals the marginal product of labour in region r.

Denoting the real investment of private capital in region r by I_t^r and the real investment of public capital in region r by $I_t^{g^r}$, private and public capital accumulate over time as follows:

$$K_{t+1}^r = I_t^r \tag{5}$$

$$G_{t+1}^r = I_t^{g^r} \tag{6}$$

It is apparent that both capital stocks depreciate completely within one period.[9]

For Foreign, the corresponding equations are as follows:

$$Y_t^* = A_t^* (K_t^*)^{\alpha} (N_t^*)^{1-\alpha} \tag{1.1}$$

$$A_t^* = A_t \left(\frac{G_t^*}{L_t^*}\right)^{\eta} \tag{2.1}$$

$$q_t^* = \alpha A_t^* \left(\frac{K_t^*}{N_t^*}\right)^{\alpha-1} \tag{3.1}$$

$$w_t^* = (1-\alpha) A_t^* \left(\frac{K_t^*}{N_t^*}\right)^{\alpha} \tag{4.1}$$

$$K_{t=1}^* = I_t^* \tag{5.1}$$

$$G_{t=1}^* = I_t^{g*} \tag{6.1}$$

2.2. Households

Two generations overlap in each period t as in the standard OLG framework of Diamond (1965). Each generation lives for two periods, working during the first when young and retiring in the second when old. Each member of the young generation supplies one unit of labour inelastically to firms. There is no labour–leisure choice.

In Home, households are differentiated by their region of residence $r = R1$, $R2$. We denote the young population residing in region r by L_t^r. In each period t, the population of each region grows according to an exogenously fixed factor Π^L. The population does not migrate between regions.[10] The young population of the less developed region, L_t^{R2}, is defined as a fixed share ξ of the young population of region $R1$; hence: $L_t^{R2} = \xi L_t^{R1}$, $\xi \geq 0$.[11] The (young)

9 Recall that one model period consists of 25–30 calendar years!

10 This assumption is motivated by differences between the regions originating in culture, language and political restrictions as currently evident within the enlarged EU.

11 Farmer and Wendner (1999: 281) used this specification for the first time.

12 Lower-case consumption quantities indicate consumption per capita. Henceforth, all lower-case quantity variables represent per-(efficiency-)capita quantities.

population of the enlarged Home is therefore given by

$$L_t = (1+\xi)L_t^{R1} \tag{7}$$

In each period of life, the young household in Home chooses between consumption of domestic commodities, $x_t^{r,n}$, $n = 1, 2$, and foreign commodities, $y_t^{r,n}$, $n = 1,2$.[12] Following Zee (1987: 605), it is assumed that only 'the domestically produced commodity can be purchased and stored by domestic residents as capital to be used in home-country production in the following period'. Real capital is therefore internationally immobile.

The budget constraint (in real and per-capita terms) of the household living in Home when young is

$$x_t^{r,1} + \left(\frac{1}{p_t}\right) y_t^{r,1} + s_t^r = (1-\tau_t)w_t^r + z_t^r \tag{8}$$

whereby

$$s_t^r \equiv \frac{K_{t+1}^{R1,r}}{L_t^r} + \frac{K_{t+1}^{R2,r}}{L_t^r} + \frac{B_{t+1}^{R1,r}}{L_t^r} + \frac{B_{t+1}^{R2,r}}{L_t^r} + \frac{(1/p_t)B_{t+1}^{*,r}}{L_t^r}$$

and when old is

$$x_{t+1}^{r,2} + \frac{1}{p_{t+1}} y_{t+1}^{r,2} = (1+i_{t+1})\left(s_t^r \equiv \frac{K_{t+1}^{R1,r}}{L_t^r} + \frac{K_{t+1}^{R2,r}}{L_t^r} + \frac{B_{t+1}^{R1,r}}{L_t^r} + \frac{B_{t+1}^{R2,r}}{L_t^r}\right) + \left(\frac{1}{p_{t+1}}\right)(1+i_{t+1}^{*,r})\frac{B_{t+1}^{*,r}}{L_t^r} \tag{9}$$

Here, p_t denotes the (external) terms of trade (units of the good produced in Foreign per unit of the good produced in Home), $i_t = (1-\varphi)q_t - 1$ is Home's and $i_t^* = (1-\varphi_t^*)q_t^* - 1$ is Foreign's real interest rate, $\tau_t(\varphi)$ is the wage (capital income) tax rate in Home, φ^* is the capital income tax rate in Foreign, and z_t^r denotes region r's real per-capita transfers from the government. Equation (8) represents region r young household's budget constraint for the working period, and it says that the consumption expenditures for Home's and Foreign's product, $x_t^{r,1} + (1/p_t)y_t^{r,1}$, plus per-capita savings in terms of Home's output, s_t^r, have to be equal to net real wage income, $(1-\tau_t)w_t^r$, plus public per-capita transfers . In addition, per-capita savings of the young household in Home's region r are used to finance the capital stock of region $R1$ ($R2$) which the household of region r plans to hold at the beginning of period, $t+1$, $K_{t+1}^{R1,r}$ ($K_{t+1}^{R2,r}$), plus the stock of domestic (foreign) government bonds which the household of region r plans to hold at the beginning of period $t+1$, B_{t+1}^r ($B_{t+1}^{*,r}$). Equation (9) denotes the budget constraint of region r young household when old, and it says that the consumption expenditures for the good produced in Home and in Foreign in terms of Home's good, $x_{t+1}^{r,2} + (1/p_{t+1})y_t^{r,2}$, are financed by capital plus interest from real capital and Home's government bonds, $(\mathbf{1}+i_{t+1})(K_{t+1}^{R1,r}/L_t^r + K_{t+1}^{R2,r}/L_t^r + B_{t+1}^{R1,r}/L_t^r + B_{t+1}^{R1,r}/L_t^r)$, as well as by capital plus interest from bonds emitted by the government in Foreign, $(\mathbf{1}/p_{t+1})(1+i_{t+1}^*)B_{t+1}^{*,r}/L_t^r$. Home households' preferences with respect to consumption goods are represented by the following inter-temporal log-linear utility function:

$$U_t^r = \zeta \ln x_t^{r,1} + (1-\zeta)\ln y_t^{r,1} + \beta\left[\zeta \ln x_{t+1}^{r,2} + (1-\zeta)\ln y_{t+1}^{r,2}\right] \tag{10}$$

where $0 < \beta < 1$ denotes the time preference factor of the young generation and $0 < \zeta < 1$ $(1 - \zeta)$ is the expenditure share for Home (Foreign) commodities. $\ln x_t^{r,1}$ $(\ln x_{t+1}^{r,2})$, respectively $\ln y_t^{r,1}$ $(\ln y_{t+1}^{r,2})$, depict typical neoclassical utility functions with positive but diminishing marginal utilities with respect to the consumption quantities of both commodities in the working and retirement periods. Each household maximizes the utility function (10) subject to the budget constraints defined by Equations (8) and (9). In solving the optimization problems it is worth noting that Home's and Foreign's government bonds are perfect substitutes within the portfolio of the young household in Home. Hence, the young household holds strictly positive quantities of both assets only if their rates of return are equal or if a real interest parity holds between Home and Foreign:

$$(1 + i_{t+1}) \left(\frac{p_{t+1}}{p_t} \right) = (1 + i_{t+1}^*) \tag{11}$$

Respecting Equation (11), the optimal consumption quantities of the domestic household are obtained from maximizing (10) subject to a reduced form of Equations (8) and (9), the so-called inter-temporal budget constraint.[13] Insertion of the optimal consumption equations into Equation (8) yields the following optimal savings function: $s_t^r = \sigma[(1 - \tau_t)w_t^r + z_t^r)$, $\sigma \equiv \beta/(1 + \beta)$.

The corresponding budget constraints and utility function for the young household in Foreign are

$$p_t x_t^{*,1} + y_t^{*,1} + s_t^* = (1 - \tau_t^*)w_t^* + z_t^* \tag{8.1}$$

where $s_t^* \equiv K_{t+1}^*/L_t^* + p_t \left(B_{t+1}^{R1,*}/L_t^* + B_{t+1}^{R2,*}/L_t^*\right) + B_{t+1}^{*,*}/L_t^*$

$$\begin{aligned} p_{t+1} x_{t+1}^{*,2} + y_{t+1}^{*,2} = {} & (1 + i_{t+1}^*) \left(K_{t+1}^*/L_t^* + B_{t+1}^{*,*}/L_t^*\right) \\ & + p_{t+1}(1 + i_{t+1}) \left(B_{t+1}^{R1,*}/L_t^* + B_{t+1}^{R2,*}/L_t^*\right) \end{aligned} \tag{9.1}$$

and

$$U_t^* = \zeta \ln x_t^{*,1} (1 - \zeta) \ln y_t^{*,1} + \beta \left[\zeta \ln x_{t+1}^{*,2} (1 - \zeta) \ln y_{t+1}^{*,2}\right] \tag{10.1}$$

It is assumed that productive capital in Home is fully mobile across regions.[14] Because of initial regional disparities within the enlarged Home, capital spontaneously flows towards the region with the higher marginal product of private capital (i.e., to region $R2$). These intra-period capital movements between $R1$ and $R2$ come to an end if the following real interest parity condition holds between $R1$ and $R2$:

$$q_t^{R1} = q_t^{R2} \tag{12}$$ [15]

2.3. The public sector

The government of enlarged Home partly finances its expenses through the emission of public debt, $B_t^{R1} + B_t^{R2}$. Public revenues include labour and capital

13 The optimal solution can be found in the appendix to Farmer and Zotti (2006: 31).

14 Complete immobility of real capital between Home and Foreign is an extreme assumption motivated by the fact that the mobility costs of real capital between countries are still significantly higher than those of financial capital.

15 Note that one period in the model comprises 25–30 years!

16 The concept of θ_t is borrowed from Funke and Strulik (2000: 367).

income taxes. The total public expenditures of region r are defined as a fixed, common share $\Gamma(0 < \Gamma < 1)$ of regional GDP X_t^R. Region-specific expenditures are allocated to transfers to private households and to expenditures for investment in public capital in region r as follows:

$$I_t^{g^r} = \gamma T X_t^r \tag{13}$$

$$L_t^r z_t^r = (1 - \gamma_t^r)\Gamma X_t^r \tag{14}$$

where $0 < \gamma_t^r < 1$, $r = R1,R2$, is the share of total expenditures spent on public capital accumulation in region r.

The budget constraint for the government of enlarged Home is therefore

$$\begin{aligned} B_{t+1}^{R1} + B_{t+1}^{R2} + \tau(w_t^{R1}L_t^{R1} + w_t^{R2}L_t^{R2}) + \varphi(q_t^{R1}K_t^{R1} + q_t^{R2}K_t^{R2}) \\ = (1 + i^t)(B_{t+1}^{R1} + B_{t+1}^{R2}) + \Gamma(X_{t+1}^{R1} + X_{t+1}^{R2}) \end{aligned} \tag{15}$$

Likewise, foreign public investment expenditures, total transfers and the budget constraint of the government in Foreign are as follows:

$$I_t^{g^*} = \gamma_t^* \Gamma Y_t^* \tag{13.1}$$

$$L_t^* z_t^* = (1 - \gamma_t^*)\Gamma Y_t^* \tag{14.1}$$

$$B_{t+1}^* + \tau_t^* w_t^* L_t^* + \varphi_t q_t^* K_t^* = (1 + i_t^*)B_t^* + \Gamma Y_t^* \tag{15.1}$$

2.4. The convergence process within Home

A new variable, the convergence indicator θ_t, is introduced to measure regional disparities within Home after enlargement[16]:

$$\theta_t \equiv \frac{x_t^{R2}}{x_t^{R1}}, \quad x_t^{R2} = \frac{x_t^{R2}}{A_t L_t^{R2}}, \quad x_t^{R1} = \frac{x_t^{R1}}{A_t L_t^{R1}} \tag{16}$$

where x_t^r, $r = R1,R2$, denotes per-efficiency-capita income in region r.

Combining the production function (1) and the real interest parity condition between region $R1$ and region $R2$ (Equation (12)) yields

$$\theta_t = \frac{x_t^{R2}}{x_t^{R1}} = \frac{k_t^{R2}}{k_t^{R1}} = \left(\frac{g_t^{R2}}{g_t^{R1}}\right)^{\frac{\eta}{1-\alpha}}, \quad k_t^r \equiv \frac{K_t^r}{A_t L_t^r}, \quad g_t^r \equiv \frac{G_t^r}{A_t L_t^r} \tag{17}$$

Equation (17) reveals that 'interregional mobile private capital ties down [$R2$'s] relative income per capita to its relative stock of infrastructure per capita' (Funke and Strulik 2000: 367).

Recalling (6), and considering (17), we obtain

$$\theta_{t+1} = \left(\frac{\gamma_t^{R2}}{\gamma_t^{R1}}\right)^{\frac{\eta}{1-\alpha}} (\theta_t)^{\frac{\eta}{1-\alpha}} \tag{18}$$

This equation depicts the motion of the convergence indicator in Home. It shows that the convergence is governed by the ratio of the regional shares of investment expenditures on public infrastructure $\gamma_t^{R2}/\gamma_t^{R1}$ and by the production elasticities of private capital α and public infrastructure η.

2.5. Inter-temporal market equilibrium

In this section, the main market clearing conditions for all periods $t = t_0$, $t_0 + 1$, $t_0 + 2$, are briefly described.

Market clearing of the regional labour markets in Home requires

$$N_t^r = L_t^r \quad r = R1,R2 \tag{19}$$

while in Foreign

$$N_t^* = L_t^* \tag{19.1}$$

holds.

In Home, full integration of the internal market for private capital services implies

$$K_t^{R1} + K_t^{R2} = K_t^{R1R1} + K_t^{R1R2} + K_t^{R2R1} + K_t^{R2R2} \tag{20}$$

The product market clearing condition of Home reads as follows:

$$\begin{aligned} x_t^{R1} + \xi x_t^{R2} = {} & x_t^{R1,1} + \left(\frac{1}{\Pi^L}\right) x_t^{R1,2} + \xi x_t^{R2,1} + \xi \left(\frac{1}{\Pi^L}\right) x_t^{R2,2} \\ & + \Pi^n g_{t+1}^{R1} + \Pi^n k_{t+1}^{R1} + \xi \Pi^n g_{t+1}^{R2} + \xi \Pi^n k_{t+1}^{R2} + x_t^{*,1} + \left(\frac{1}{\Pi^L}\right) x_t^{*,2} \end{aligned}$$

$$\Pi^n \equiv \Pi^L \Pi^A \tag{21}$$

while foreign product market clearing demands

$$\begin{aligned} y_t^* = {} & y_t^{*,1} + \left(\frac{1}{\Pi^L}\right) y_t^{*,2} + \Pi^n k_{t+1}^* + \Pi^n g_{t+1}^* + y_t^{R1,1} + \left(\frac{1}{\Pi^L}\right) y_t^{R1,2} \\ & + \xi y_t^{R2,1} + \xi \left(\frac{1}{\Pi^L}\right) y_t^{R2,2} \end{aligned} \tag{21.1}$$

The world market for Home bonds clears according to

$$B_t^{R1} + B_t^{R2} = (B_t^{R1,R1} + B_t^{R1,R2} + B_t^{R1,*}) + (B_t^{R2,R1} + B_t^{R2,R2} + B_t^{R2,*}) \tag{22}$$

and symmetrically for Foreign bonds we have

$$B_t^* = (B_t^{*,R1} + B_t^{*,R2} + B_t^{*,*}) \tag{21.1}$$

The world capital market clearing condition requires that the total amount of per-capita savings in the world equals the world per-capita demand for loanable funds:

$$s_t^{R1} + \xi s_t^{R2} + \left(\frac{1}{p_t}\right) s_t^* = \Pi^n \left[k_{t+1}^{R1} + \xi k_{t+1}^{R2} + (1+\xi) b_{t+1}^{R1} + \left(\frac{1}{p_t}\right) k_{t+1}^* + \left(\frac{1}{p_t}\right) b_{t+1}^* \right] \tag{23}$$

17 Bianconi (2003: 29) considers a similar symmetric equilibrium in an infinitely lived agent (ILA) framework.

18 The international equality of public capitals per capita and income tax rates ensures terms of trade of unity (symmetric steady state). These simplifying assumptions are warranted in an age of global intergovernmental competition.

19 To simplify the cumbersome notation note that in the dynamical equations the superscripts indicating region $R1$ of Home are henceforth deleted.

3. THE INTER-TEMPORAL EQUILIBRIUM DYNAMICS UNDER TIME-STATIONARY PUBLIC POLICIES AND THE SYMMETRIC STEADY STATE

This section is devoted to presenting the inter-temporal equilibrium dynamics of the two-country, two-region model described in the previous section and to studying steady states with pre-shock terms of trade equal to unity (symmetric equilibrium in terms of relative output prices).[17]

From the model description above, it transpires that the inter-temporal equilibrium dynamics of the endogenous variables depend on the path of the policy instruments of governments in Home and Foreign. As the optimization of governments over policy instruments is beyond the scope of this article, we have to specify how both governments determine their inter-temporal policy profile. In accordance with a large part of the established literature on growth and government debt (Diamond 1965; Azariadis 1993), we introduce the following assumptions A1, A2 and A3 on how Home and Foreign governments determine the inter-temporal fiscal policy profiles:

A1 (*Constant-stock policies*). Home and Foreign governments stick to a policy of holding the stocks of government debt per capita and public capital per capita constant over time.

A2 (*Time-stationary capital income tax rates*) Home and Foreign governments fix time-stationary ('constant') capital income tax rates.

A3 (*Internationally identical capital income tax rates, total public expenditure ratios and public capitals per capita*). Home and Foreign governments specify identical income tax rates, identical total public expenditure ratios and identical public capitals per capita.[18]

Additionally, we make the following simplifying assumption: A4 (*Equality of population size of pre-enlarged Home and Foreign*). In symbols, $L_t = L_t^*$ (A4) does not alter the qualitative nature of the results.[19]

From the international interest parity condition (11) together with assumptions A1–A4, the equation of motion of the terms of trade follows:

$$p_{t+1} = \frac{p_t[(k_{t+1}^*)^{\alpha-1}]}{(k_{t+1})^{\alpha-1}} \tag{24}$$

To ensure perfect convergence in Home in the long run, A5 (*Equality of long-run public debts per capita in both regions of Home*). $b \equiv b^{R1} = b^{R2}$, is presupposed. The budget constraints of Home and Foreign governments (see Equations (15) and (15.1)) imply that tax rates become endogenous. Applying the assumptions introduced so far, domestic and foreign wage tax rates, τ_t, τ_t^*, are determined as follows:

$$\tau_t = \frac{(1/\Pi^n)(g)^\eta(k_t)^\alpha\,[\alpha(1-\varphi)(1+\xi)(b/k_t) + (\Gamma - \alpha\varphi)(1+\xi\theta_t)] - (1+\xi)b}{(1-\alpha)(1+\xi\theta_t)(1/\Pi^n)(g)^\eta(k_t)^\alpha} \tag{25}$$

$$\tau_t^* = \frac{(1/\Pi^n)(g)^\eta(k_t^*)^\alpha[\alpha(1-\varphi)(b^*/k_t^*) + \Gamma - \alpha\varphi] - b^*}{(1-\alpha)(1/\Pi^n)(g)^\eta(k_t^*)^\alpha} \tag{25.1}$$

From the equation of motion for the public capital stock per capita (6) and A1, Home's (Foreign's) share of investment expenditures on public

infrastructure becomes endogenous and it follows from (6), respectively (6.1), that

$$\gamma_t = \frac{\Pi^n}{\Gamma}\frac{g^{1-\eta}}{(k_t)^{\alpha}} \tag{26}$$

$$\gamma_t^* = \frac{\Pi^n}{\Gamma}\frac{g^{1-\eta}}{(k_t^*)^{\alpha}} \tag{26.1}$$

By inserting the optimal savings function of Home from the previous section and the corresponding Foreign savings function into the world capital market clearing condition (23), respecting profit maximizing conditions (3) and (3.1) as well as the equations for wage tax rates (25) and public investment expenditure ratios (26), the following difference equation is obtained:

$$\begin{aligned}(1+\xi\theta_{t+1})p_t k_{t+1} &= (1+\xi\theta_t)p_t[\sigma_0(k_t)^{\alpha} - \sigma g] - (1+\xi)p_t b \\ &\quad \times \left\{\sigma\left[\frac{(1+i_t)}{\Pi^n} - 1\right] + 1\right\} + \left[\sigma_0(k_t^*)^{\alpha} - \sigma g\right] - b^*\left\{\sigma\left[\frac{(1+i_t)}{\Pi^n} - 1\right] + 1\right\}\end{aligned}$$

$$\sigma_0 \equiv \sigma\left(\frac{1}{\Pi^n}\right)(g)^{\eta}[1-\alpha(1-\varphi)] \quad i_t \equiv \alpha(1-\varphi)(g)^{\eta}(k_t)^{\alpha-1} - 1$$

$$i_t^* \equiv \alpha(1-\varphi)(g)^{\eta}(k_t^*)^{\alpha-1} - 1 \tag{27}$$

From the two national product market clearing conditions (21) and (21.1), the third dynamic equation is obtained:

$$\begin{aligned}k_{t+1}^* - \left[\frac{(1-\zeta)}{\zeta}\right](1+\xi\theta_{t+1})p_t k_{t+1} &= \left[\left(\frac{1}{\Pi^n}\right)(g)^{\eta}(k_t^*)^{\alpha} - g\right] \\ &\quad - \frac{(1-\zeta)}{\zeta}(1+\xi\theta_t)p_t\left[\left(\frac{1}{\Pi^n}\right)(g)^{\eta}(k_t)^{\alpha} - g\right]\end{aligned} \tag{28}$$

The fourth equation of the inter-temporal equilibrium dynamics is identical to Equation (18) and simplifies under the assumption A6 (*Interregional equality of public investment expenditure rations in Home*), i.e., $\gamma_t^{R2} = \gamma_t^{R2} = \gamma_t, \forall t,$[20] to

$$\theta_{t+1} = (\theta_t)^{\frac{\eta}{(1-\alpha)}} \tag{29}$$

Equations (24, 27–29) represent the four-dimensional dynamic system of the two-country, two-region model of the world economy.

The first step to analyse this dynamic system is to ask for the existence of fixed points of the equilibrium dynamics or steady-state solutions, i.e., $k_{t+1} = k_t = k \quad k_{t+1}^* = k_t^* = k^* \quad p_{t+1} = p_t = p \quad \theta_{t+1} = \theta_t = \theta$.

It is easy to see that at a fixed point of the dynamical system (24), (27–29) the endogenous variables are determined as follows: the steady-state convergence indicator is $\theta = 1$, steady-state Foreign private capital per capita is $k^* = k$, steady-state terms of trade of Home are $p = [\zeta/\left((1-\zeta)(1+\xi)\right)]$ and steady-state private capital per capita in Home equals the values of k, which solves

$$k = F(k) \equiv \sigma_0 k^{\alpha} - \sigma g - \vartheta\left\{\sigma\left[\frac{(1+i)}{\Pi^n} - 1\right] + 1\right\}$$

20 At first sight, this assumption appears extremely restrictive. However, applying the model to the EU, it is less so because Eastern European countries are obliged to follow the fiscal criteria of the Maastricht treaty, which imply strict upper bounds for government expenditures similar to those of EU-15.

with ϑ representing the weighted average of per-capita public debt in Home and in Foreign, $\vartheta \equiv \zeta b + b^*(1-\zeta)$, where the weights are the expenditure shares of Home's young households for Home's and Foreign's products.

In general, it is not sure that there exist strictly positive (non-trivial) values of k which solve the equation $k = F(k)$. However, if $0 < \vartheta$ is bounded from above by a finite parameter value $\bar{\vartheta}$, i.e., $\vartheta \in (0,\bar{\vartheta})$, then there exist two non-trivial[21] steady-state per-(efficiency-)capita capital stocks $(0 < k^L < k^H < \bar{k})$.[22] In other words: if public debt in Home and Foreign is not 'too large', two non-trivial steady-state solutions of the capital intensity in Home exist. Figure 1 illustrates these multiple fixed-point or steady-state solutions for the capital intensity in Home.

21 In fact, there is also a trivial steady-state solution. However, in contrast to intuition, the existence of a trivial steady state cannot be inferred from an inspection of the steady-state lines. To see how the existence of a trivial steady-state solution can be demonstrated in a similar formal context, see Farmer and Wendner (2003: 780).

22 k equal to $\bar{k}$ implies zero consumption in steady state.

4. STABILITY OF STEADY STATES AND THE LONG-RUN EFFECTS OF ENLARGEMENT

Being assured of the existence of two distinct steady-state solutions, it is natural to ask about the local stability of the steady states. To this end, the equilibrium dynamics is linearly approximated in a small neighbourhood of each of the two steady states. The Jacobian of the dynamic system (24), (27–29) is written as usual[23]:

$$J = \begin{bmatrix} \partial p_{t+1}/\partial p_t & \partial p_{t+1}/\partial k_t & \partial p_{t+1}/\partial k_t^* & \partial p_{t+1}/\partial \theta_t \\ \partial k_{t+1}/\partial p_t & \partial k_{t+1}/\partial k_t & \partial k_{t+1}/\partial k_t^* & \partial k_{t+1}/\partial \theta_t \\ \partial k_{t+1}^*/\partial p_t & \partial k_{t+1}^*/\partial k_t & \partial k_{t+1}^*/\partial k_t^* & \partial k_{t+1}^*/\partial \theta_t \\ 0 & 0 & 0 & \partial \theta_{t+1}/\partial \theta_t \end{bmatrix} \tag{30}$$

23 The calculation of the elements of the Jacobian can be found in the appendix to Farmer and Zotti (2006: 32–33).

One can show that both the determinant as well as the trace of the Jacobian matrix (30) evaluated at a non-trivial steady state are strictly larger than zero. It is also possible to prove mathematically that all characteristic values of the Jacobian matrix (30) are real and strictly larger than zero and, depending on whether the characteristic values are evaluated at the lower or the higher capital intensity, two or one characteristic value is larger than unity. In the former case,

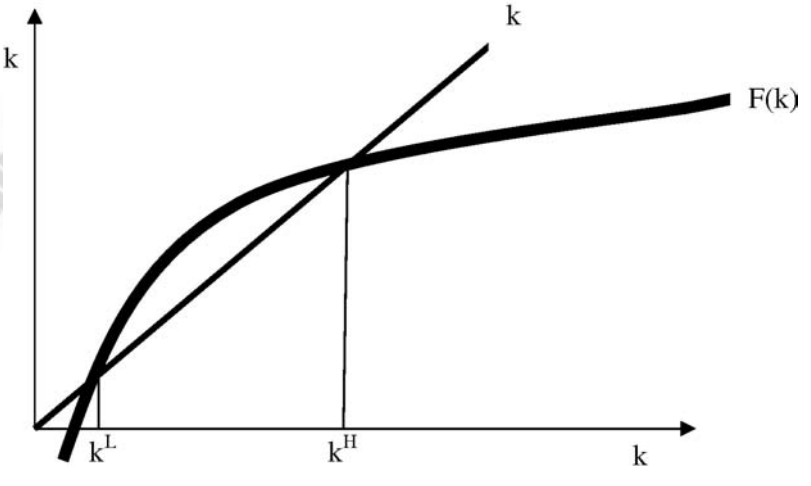

Figure 1: The existence of two steady-state solutions.

the steady-state solution is unstable which means that starting from a non-steady-state value of Home's capital intensity there is no trajectory emanating from the initial value of k which ends in k^{L}. In the latter case, there exists a so-called stable manifold in the four-dimensional space of dynamic variables along which the dynamic system converges to k^{H}. However, there is one and only one initial value of the terms of trade associated with the predetermined values of Home and Foreign capital intensity and of the convergence indicator from which the dynamic system can start. If this initial value of the terms of trade of Home is missed, the dynamic system strays in finite time. In other words, the dynamic system in the neighbourhood of the higher capital intensity is only saddle-path or knife-edge stable, a property typical for linear dynamic economic systems with rational expectations (Blanchard and Kahn 1980).

Knowing which steady state qualifies as being locally saddle-point stable, we can now turn to the investigation of the long-run effects of the enlargement of Home upon the main variables of the two-country, two-region model of the world economy. In the long run, full convergence between both regions of Home is achieved ($\theta = 1$). We compare the steady-state solution of the two-country model when Home consists only of region $R1$ while region $R2$ remains in autarky ($\xi = 0$) with the steady state when region $R2$ is fully integrated into the world economy ($d\xi = \xi > 0$). Remembering constant returns to all factors in both regions and in both countries, one would expect that per-(efficiency-) capita variables in the long run do not respond to the increase of Home population. Recalling from the previous section that in a full-convergence steady state $k^{*} = k$ and $k = F(k)$ hold, where $F(k)$ is independent of parameter ξ, the expectation is easily verified.

On the other hand, since in full-convergence steady state $p = \zeta/[(1 - \zeta)(1 + \xi)]$ holds, it is immediate that the terms of trade of Home deteriorate when ξ increases as ζ does not change. Intuitively, Home's terms of trade worsen as the relative scarcity of the foreign product rises with a larger supply for consumers in Home's commodity market and an unchanging supply in the commodity market of Foreign. Consequently, Home's trade balance and current account will improve, because in the two-country, two-region model, the so-called Marshall–Lerner conditions hold which state that if export and import demands respond elastically to the terms of trade, Home's trade balance and current account will improve if the terms of trade worsen.

The basic insight of neoclassical growth theory that the population size of countries does not matter for the determination of per-capita income in the long run is verified. On the other hand, the international competitiveness of the enlarged economic union in comparison with the not enlarged Foreign is affected by the enlargement shock: contrary to political rhetoric, the international competitiveness does not rise but declines, because the terms of trade of the enlarged Home deteriorate by the factor $p/(1 + \xi)$.

The result of diminishing international competitiveness of enlarged Home and rising international competitiveness of Foreign is also corroborated by the fact that steady-state welfare of Home's household declines while steady-state welfare of the household in Foreign increases. This result is most easily explained by having a look at Equations (3), (4), (8), (9) and (25) evaluated at the full-convergence steady state. As Home's capital intensity k is independent of ξ, real capital service price and the real wage rate in region r also do not respond to the enlargement shock. For steady state k and $\theta = 1$, it transpires from Equation (25) that Home's wage tax rate is also independent of ξ. As is well known, maximization of a log-linear inter-temporal utility function such

as (10) subject to (8) and (9) implies that the expenditures for each good are equal to a fixed proportion of net wage income which does not respond to the enlargement shock. Thus, because both $(1/p)y^{r,1}$ and $(1/p)y^{r,2}$ are fixed and p decreases, it follows immediately that $y^{r,1}$, and $y^{r,2}$ have to decrease. $x^{r,1}$ and $x^{r,2}$ are determined by net wage income and thus are also independent of ξ. Hence, the inter-temporal utility function (10) implies unambiguously that the inter-temporal utility (lifetime welfare) of the household in Home declines. Similar reasoning corroborates rising lifetime welfare for the household in Foreign.

The steady-state effects of $R1$ enlargement are clear cut, because in the long run any convergence effect does not arise. The per-capita income of region $R2$ has by assumption been fully converged to that of region $R1$. Along the transition path towards the post-enlargement steady state with perfectly converged Home regions, the enlargement effects are not so easily understood. In this case an initial non-steady-state situation has to be compared with a full-convergence steady state and a transition process arises.[24] Enlargement of developed region $R1$ by the less developed region $R2$ thus has two distinct effects: an integration effect and an accompanying convergence effect. As we already know, the former includes the effects of the integration of the capital and good markets of region $R2$ into the respective markets of region $R1$ (see Equations (20) and (21))[25] as well as the integration of the capital market of region $R2$ into the world capital market (see Equation (23)).[26] The latter effect consists of faster growth of the economy of region $R2$ due to free capital movements.

In the subsequent sections both the integration and convergence effects of the enlargement of highly developed Home by a less developed region are investigated. First, the economic rationale behind the instantaneous jump of the terms of trade onto the stable manifold of the equilibrium dynamics is considered. Second, the subsequent motion of all dynamical variables along the stable manifold (transitional dynamics) is subject to scrutiny.

24 To the best of the author's knowledge there is hardly any work in inter-temporal macroeconomics (see Azariadis (1993) and more recently de la Croix and Michelle (2002) for excellent overviews) which investigates the transition from a non-steady-state to a steady-state situation. Without exception, the transition from a former to a new steady state is analysed.

25 From this point of view, enlargement of $R1$ is a typical regional integration shock, accounting for three of the four single-market effects (that of the free movement of people is not taken into consideration; see Breuss (2002)).

26 This assumption also allows us to model partial globalization effects (see Breuss 1997) of the enlargement.

5. THE ENLARGEMENT SHOCK AND ITS SHORT-RUN EFFECTS ON TERMS OF TRADE AND PRIVATE CAPITAL IN HOME AND FOREIGN

In order to be able to investigate the transitional dynamics of the terms of trade and private capital formation in Home and Foreign following Home's heterogeneous enlargement, the type and the timing of the enlargement shock are to be specified. To keep the analysis as simple as possible, we assume that the enlargement shock is permanent, is unannounced and occurs at the beginning of the transition period. As mentioned above, the value of the parameter ξ increases suddenly from zero to a strictly positive value. The period in which the shock is introduced is denoted by $t = t_0$ and it is called the enlargement period.

It makes a difference whether the enlargement takes place after the conclusion of the convergence process or before. In the first case, the terms of trade jump instantaneously to its new steady-state value. Only in the latter case do the terms of trade move along a transition path towards the new steady state as shown below.

Along the transition path of the terms of trade, two phases can be distinguished: first the instantaneous jump of the terms of trade onto the stable manifold of the transitional dynamics and then their motion along the stable manifold towards the steady state of full convergence.

Without going into details of the mathematical derivation of the stable manifold,[27] two observations concerning the nature and main properties of the stable manifold are in order. First, the terms of trade in the enlargement period can be written as a linear combination of the new steady-state value of the terms of trade and the deviation of the convergence indicator from unity. Second, the constant coefficients in front of steady-state terms of trade and in front of the deviation of the convergence indicator from unity are positive. Taken together, these observations have two implications: first, the terms of trade in the enlargement period do not 'overshoot' (Dornbusch 1976) the new steady-state value, and second, a transitional dynamics of the terms of trade arises only if a highly developed region is enlarged by a less developed region, i.e. $\theta_{t_0} < 1$. If region $R2$ were equally developed, $\theta_{t_0} = 1$, no transitional dynamics would occur.

To explain the economic rationale behind the immediate decline of the terms of trade in the enlargement period, we assume on the contrary that the terms of trade do not respond to the enlargement shock. Under this proviso, imbalances in the domestic and foreign balance of payments in the enlargement period will occur, which is inconsistent with an inter-temporal general equilibrium. To simplify as much as possible the following explanation without losing generality, we further assume that per-capita public debt in Home is zero: $b = 0$.[28] Under this assumption Equation (25) implies that the wage tax rate in Home is independent of ξ. Moreover, per-capita capital stocks in Home, k_{t_0}, and Foreign, $k^*_{t_0}$, do not adapt to the enlargement shock as they are fixed by the investment decisions of the previous period. In view of the first-order conditions for profit maximization (3) and (4) in Home, respectively (3.1) and (4.1) in Foreign, real wages, real capital prices and real interest rates in the enlargement period neither change in Home nor in Foreign. Equations (1) and (2), respectively (1.1) and (2.1), imply that per-capita outputs in Home and Foreign also remain unchanged. As neither net wage nor interest income in Home and Foreign responds to the enlargement shock, per-capita consumption of the domestic good by Home's young households, $x^{r,2}_{t_0}$, $r = R1,R2$, and by Home's old households, $x^{r,2}_{t_0}$, $r = R1,R2$, as well as per-capita consumption of the domestic good by Foreign's young households, $y^{r,1}_{t_0}$, and by Foreign's old households, $y^{r,2}_{t_0}$, remain unchanged. Because by assumption p_{t_0} does not respond to the enlargement shock, it also follows that Home's export quantities, $x^{*,1}_{t_0}$ and $x^{*,2}_{t_0}$, do not adapt to the enlargement shock. Likewise, per-capita imports of the foreign good by Home's young and old households, $y^{r,1}_{t_0}$, $r = R1,R2$, respectively $y^{r,2}_{t_0}$, $r = R1,R2$, remain unchanged. However, Home's trade balance, defined as

$$x^{*,1}_{t_0} + \left(\frac{1}{\Pi^L}\right) x^{*,2}_{t_0} - \left(1/p_{t_0}\right)\left[y^{R1,1}_{t_0} + \left(\frac{1}{\Pi^L}\right) y^{R1,2}_{t_0} + \xi y^{R2,1}_{t_0} + \xi\left(\frac{1}{\Pi^L}\right) y^{R2,2}_{t_0}\right]$$

deteriorates, because through enlargement Home's imports increase by $\xi y^{R2,1}_{t_0} + \xi(1/\Pi^L) y^{R2,2}_{t_0}$.

The current account defined as trade balance plus interest income from foreign government bonds held by Home's residents becomes definitively negative[29] if it is assumed for simplicity that $R2$ households do not hold foreign government bonds. As a sustained non-negative current account is incompatible with the new steady state, the assumption that the terms of trade in the shock period do not respond to the enlargement shock cannot be maintained

27 See again Farmer and Zotti (2006: 33–34) for mathematical details.

28 Note however that the qualitative results obtained in this subsection hold also in the general case of a non-vanishing domestic public debt per capita as numerical calculations show.

29 Note that in steady state current account is balanced.

any longer. Hence, 0*tp* has to change and the next question is why the terms of trade jump downwards.

The clue to answering this question is whether the deficit of Home's current account can be removed by worsening the terms of trade or not. To begin with, consider first Home's exports. From the solution of intertemporal utility maximization problem of the young household in Foreign (see (10.1) subject to (8.1) and (9.1)), it is immediate that exports unambiguously increase if terms of trade decrease while the net wage and the interest rate do not change in Foreign. It is also true that Home's import quantities from abroad decline when the terms of trade worsen. In view of the definition of the trade balance, what is above everything is how the real import value (import quantities multiplied by the inverse of the terms of trade) in comparison to export quantities responds to worsening terms of trade. The trade balance and hence the current account improve if the real import value increases less than export does with declining terms of trade, which is equivalent to the claim that export and import quantities react elastically to changes in the terms of trade (Marshall–Lerner condition). As the terms-of-trade elasticities of export and import functions derived from log-linear utility functions are unity, the Marshall–Lerner condition is fulfilled and hence the downward jump of the terms of trade improves Home's current account.

6. HOME'S REGIONAL CONVERGENCE AND THE TRANSITIONAL DYNAMICS OF THE WORLD ECONOMY

This section focuses on the description of the convergence effects after the enlargement period. While the economic integration of region *R*2 into the world economy is achieved within the enlargement period, the convergence process within Home is perfectly completed only after an infinite number of periods. In practice, the convergence is roughly completed after several model periods (approximately 60–80 calendar years). Convergence proceeds monotonically because it is governed by the dynamic motion of the monotonic convergence indicator (see Equation (29)).

The international effects of the convergence process are reflected by the transitional dynamics of the terms of trade after its instantaneous downward jump and by the dynamics of private per-capita capital stocks in Home and Foreign. We start with an analysis of the terms-of-trade dynamics, which is governed by Equation (24). The main question is whether the terms-of-trade dynamics is monotonous or not. As it is obvious from Equation (24), the answer depends mainly on the relationship between Home's and Foreign's capital intensities in period $t+1$ and period-t terms of trade.

Upon solving Equations (27) and (28) simultaneously for k_{t+1} and k^*_{t+1} and differentiating the resulting equations with respect to p_t, one can show that $\partial k_{t+1}/\partial p_t > 0$ and $\partial k^*_{t+1}/\partial p_t < 0$ hold, or in other words, there is a positive (negative) relationship between Home's (Foreign's) $t+1$ capital intensity and period-t terms of trade. Intuitively, Home's capital accumulation declines over time with worsening terms of trade because Home's rising exports crowd out private capital formation in both regions of Home (see market clearing condition (21)) while capital accumulation in Foreign increases with worsening terms of trade because declining imports of Home crowd in private capital formation in Foreign (see market clearing condition (21.1)).

Knowing the relationship between Home's and Foreign's capital intensities in period $t+1$ and period-t terms of trade, it is easy to see from Equation (24) that the dynamics of the terms of trade is monotonous. As $\alpha < 1$, p_{t+1} rises with higher k_{t+1} and lower k^*_{t+1}. Hence, $\mathrm{d}p_{t+1}/\mathrm{d}p_t > 0$, which means that the dynamics of the terms of trade is monotonous, or in other words that the terms of trade are worsening along the transition path towards the new steady state when they are deteriorating in the enlargement period.

Next, it is required to explain intuitively the response of private per-capita capital stocks k_{t_0+1} and $k^*_{t_0+1}$ to the enlargement shock. The total reaction of private capital intensities in Home and Foreign can be seen as the result of two partial responses to the enlargement shock: the first partial response is defined as change of Home and Foreign capital intensities under the proviso of unchanged terms of trade, while the second partial response is seen as reaction of capital intensities to changing terms of trade p_{t_0}.

As regards the former, the per-capita capital stock k_{t_0+1} increases because in the domestic product market (see Equation (21)) production supply increases (through the number of suppliers from region $R2$) and this increase is always larger than the increase in consumption demand (through the number of demanders from region $R2$), a consequence of the fact that a portion of region's $R2$ product (income) must be left over for the purchase of Foreign commodities. Assuming unchanged terms of trade, the per-capita capital stock $k^*_{t_0+1}$ in Foreign decreases strongly as the product supply in Foreign does not change and hence private capital in Foreign is crowded out by the larger number of Home importers of Foreign's good.

As regards the latter, it is already known from above that private per-capita capital stock in Home decreases and Foreign's per-capita capital stock increases when the terms of trade decrease.

The total effect of the enlargement shock on private capital accumulation in Home and Foreign cannot unambiguously be evaluated because the partial effects are counteracting. Hence, a numerical specification of the model parameters is needed to be able to evaluate the total effect definitely. For all admissible parameters which imply the existence and saddle-point stability of the higher steady state, one can show that a rising ξ lets both per-capita capital stocks in Home and Foreign decline. Moreover, upon solving Equations (27) and (28) simultaneously for k_{t_0+1} and $k^*_{t_0+1}$, and then partially differentiating the solutions for both capital intensities with respect to p_{t_0} and ξ, it becomes visible that the capital intensity in Home declines for another reason than the capital intensity in Foreign. While the capital intensity in Home declines because the indirect effect of the enlargement shock via the terms of trade overwhelms the direct effect of a larger ξ on k_{t_0+1}, the opposite is true as regards the decline of the capital intensity in Foreign. Moreover, the decrease of the capital intensity of Home's region $R1$ is quantitatively larger than the decrease of the capital intensity in Foreign. As can be seen from Figure 2,[30] private capital in region $R1$ experiences a relatively sharp decline in the first period after the shock and starts increasing from the second period onwards. In Foreign, capital intensity initially decreases slightly (almost not visible in Figure 2) but for a longer time span. Afterwards, it starts increasing again towards its steady-state level.

$R2$'s capital intensity faces a monotonic upward trend over the whole convergence period, until meeting the long-term level of region $R1$.

30 Clearly, Figure 2 presupposes certain numerical values of structural parameters which are specified as described in the following. Empirically, the yearly growth rate of the world active population is around 0.54%. The corresponding growth factor, G^L, over a 25 years' period is 1.1441. The utility discount factor, β, is set equal to 0.6. This figure is equivalent to a subjective time preference rate of 0.02 per year (see Auerbach and Kotlikoff (1987: 51) for a similar figure). The scale parameter A is set equal to 5.0 (Auerbach and Kotlikoff (1998: 260) assume $A = 10$ for a two-generation model of a closed economy). Under perfect competition, the production elasticity of labour $(1-\alpha)$ corresponds to the wage share, which is roughly equal to two thirds in developed countries. Therefore, $\alpha = 0.35$ is chosen. The production elasticity of public capital η is set equal to 0.3 as in Funke and Strulik (2000: 372).

31 As Equation (25) shows, the lower per-capita income has multiple, counteracting effects on the wage tax rate. From numerical analysis, it emerges that the overall effect is negative.

Moving on to the description of the evolution of other main model variables, it can be seen from Equation (1) that in period $t_0 + 1$ the (per-capita) product of Home's region $R1$, x_{t_0+1}, is also lower. On the other hand, the $R1$ wage tax rate increases.[31] As a consequence, the net incomes of the young household in $R1$ decrease. The consumption demands of the young and old households in region $R1$ decrease. The same holds true for $R1$ savings. Similar considerations apply to the foreign economy, albeit with changes of much smaller magnitude. In the converging region $R2$, private and public capital stocks are higher than in the enlargement period, allowing higher (per-capita) output levels. This fact reverberates positively on the wage income, letting consumption and savings increase accordingly.

In the product market in Foreign, the overall change of the aggregate consumption demand is, in general, indeterminate. Numerical calculations for all admissible parameter values show that it declines. The aggregate supply in Foreign decreases as well. The private investment demand also declines because the supply decrease exceeds the demand decrease.

In the product market in Home, the overall supply increases due to the strong increase of the per-capita output in region $R2$. On the demand side, in contrast to the product market in Foreign, the aggregate consumption demand rises. In addition, public and private investment expenditures in region $R2$ also increase. As the rise of the aggregate supply is larger than the demand increase, private capital investment in region $R1$ rises. Here, the initial decline of the private capital intensity in region $R1$ is reversed.

Beginning with period $t_0 + 2$, the private capital intensity in $R1$ keeps on growing monotonically towards the steady state. The dynamics of the private capital intensity in region $R2$ follows the law of motion of both the private capital stock in region $R1$ and the convergence indicator (see Equation (29)). However, in contrast to the capital intensity in region $R1$, there is no decline of $R2$ capital intensity in $t_0 + 1$ because the increase of the convergence indicator exceeds the decline of the capital intensity in region $R1$. Hence, the private capital intensity in $R2$ increases monotonically from the beginning towards the steady state, thus lowering marginal productivity and hence the real rate of return on capital in $R2$, and decreasing the incentive to invest in region $R2$.

The transition path of private capital in Foreign is affected differently from that of the domestic region $R1$. As mentioned above, the capital stock in Foreign decreases muss less than in $R1$ (almost not visible in Figure 2), but the fall persists longer.

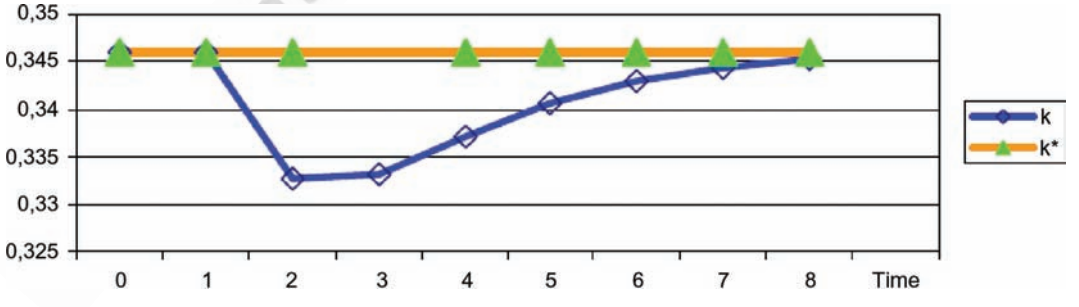

Figure 2: Transitional dynamics of domestic and foreign capital intensities.
Source: Own calculations.

7. CONCLUSIONS

This article presents a model-based answer to the question of how 'heterogeneous enlargement' of an economic union and the following intra-union economic convergence impact on its international competitiveness. The model used represents an extension of Zee's (1987) two-country OLG model by splitting up Home into two regions with initially hugely differing per-capita incomes and by introducing a two-region version of Barro's (1990) endogenous growth engine. Although highly stylized, the two-country, two-region model is still capable of accounting for the effects of enlargement of a highly developed economic area by a less-developed region and the impacts of the subsequent convergence process on the economic relations between enlarged Home and another large open economy named Foreign. The analysis focuses on the dynamics of the terms of trade between Home and Foreign, the main competitiveness indicator of enlarged Home.

The inter-temporal equilibrium dynamics of the two-country, two-region model is analytically presented and solved for non-trivial steady-state solutions. The existence, uniqueness and local stability of the steady-state solutions are stated without formal proofs. Being assured of the existence of a saddle-path stable steady state, the long-run and transitional dynamics effects of the integration of the less developed region $R2$ into the more developed two-country world economy are then economically explained.

In the long run, the terms of trade, the trade balance, the current account and the welfare of the households in enlarged Home and the per-capita variables of the less developed region of Home are affected by the enlargement, while the pre-integration Home and Foreign private per-capita capital stocks remain unaltered. While the terms of trade of Home deteriorate and the welfare of Home's households decline, Home's trade balance and current account improve and the welfare of the household in Foreign increases.

In the medium run, the enlargement brings about integration as well as convergence effects. Along the transition path of the convergence of the domestic regions, the per-capita income of the less developed region $R2$ increases monotonically from its initial level to full convergence, with most of the convergence being achieved several periods after the enlargement. The terms of trade of enlarged Home, being the sole jump variable of the dynamic system, adapt to the enlargement shock of integrating a ξ percentage of region $R2$'s population into Home by falling significantly. The terms of trade continue to fall during the entire catching-up process of region $R2$.

Private capital in region $R1$ experiences a relatively sharp decline in the first period after the shock and starts increasing from the second period onwards. $R2$'s capital intensity faces a monotonic upward trend over the whole convergence period, until meeting the long-term level of region $R1$. In Foreign, capital intensity initially decreases slightly but for a longer time span. Afterwards, it starts increasing again towards its old steady-state level.

Functional specification of preferences and technologies and the assumption of identical technologies across countries have both merits and limitations. As regards the former, closed-form solutions of the already complex dynamics of the two-country, two-region OLG model are obtained, which is impossible in dynamic equilibrium models with general utility and production functions. In view of empirical applications à la CGE models in the tradition of Auerbach and Kotlikoff (1987), it is no minor advantage to know exactly the range of model parameters which ensure existence and dynamic stability of the theoretical

core of a full-blown CGE model of the world economy comprising multiple countries of different sizes, regions and production technologies. On the other hand, it is clearly desirable to have existence and stability propositions for two-country OLG models with more general utility and production functions, which in particular differ across countries. As we know so far, while existence and stability propositions are still possible, analytical representations of the transitional dynamics become extremely cumbersome. Future research has to clarify how much can be accomplished along the analytical route and when numerical simulations become appropriate.

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